SCHOOL SCIENCE AND MATHEMATICS

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ON TO CHICAGO

Plan now to meet your fellow teachers of science and mathematics at the Edgewater Beach Hotel in Chicago on November 28 and 29 at the Annual Convention of C.A.S.M.T. Take a look at the program of the general sessions. Percy L. Julian of the Glidden Company, Chicago, will speak on "Today's Science and Our Basic Philosophies." Dr. Julian has received praise from scientists all over the world for his research discoveries. He is a former teacher and now a director of research for an industrial firm. One of his recent discoveries is a new process for isolating and preparing commercially soybean protein. He is a distinguished orator who is much in demand.

When Professor Samuel K. Allison, Director of the Institute of Nuclear Studies, University of Chicago, learned of our need for a speaker to discuss the peace time use of atomic energy, he agreed to accept the assignment himself. He is an outstanding scientist and an excellent speaker. Raleigh Schorling of the University of Michigan represents the mathematics teachers at the general sessions. During the war and since that time Mr. Schorling has served on most of the major committees concerned with the teaching of science and mathematics. He has served as chairman of several of them and has made his influence felt throughout the country. Dr. Andrew C. Ivy, Vice-President of the University of Illinois, in charge of the Professional Schools at Chicago, will discuss "Some Ethical Implications of Science." Dr. Ivy was the chief expert witness for the prosecution before the American military tribunal that convicted and sentenced seven Nazi physicians. Recently he was awarded the gold key of the American Congress of Physical Medicine for his research studies.

Don't you agree that our members have a treat in store? If you examine the section programs, you will find them just as interesting. Section Officers and Association members have assembled outstanding talent to discuss timely topics. See the October issue of this JOURNAL for more details on the program. Consult the "Yearbook" for the complete program. It will be mailed to all members about November 1st.

By popular request from members, the officers have arranged for both a Luncheon and Banquet meeting on Friday. Radio Corporation of America is furnishing the luncheon program in the famed Marine Dining Room, and Westinghouse Electric Corporation is furnishing the evening program at the banquet. For Saturday afternoon a local trip committee has arranged for a special lecture-demonstration at the Adler Planetarium for our members and also for conducted tours at the Museum of Science and Industry.

Hotel rooms are still very scarce. Make your reservations now by writing directly to: Mr. John L. Nelson, Reservation Office, Edgewater Beach Hotel, Chicago, Illinois, and mention the Central Association.

GEORGE E. HAWKINS, *President*

J. C. FITTERER

Prof. J. C. Fitterer of the Mathematics Department of the Colorado School of Mines at Golden, Colorado died after a short illness on March 12, 1947. He has been an enthusiastic reader of this journal since 1932 and a member of the Central Association of Science and Mathematics Teachers for the past few years.

HYBRID TREES HASTEN REFORESTATION WORK

Hybrid trees may bring about faster reforestation. It takes hybrid trees only from one-half to one-third the time required for a good nonhybrid timber tree to reach the same size, U. S. Department of Agriculture scientists have found.

Already a hybrid pine has been developed that at three years is more than twice as high and three times as heavy as the better of its two parents. It will take years to make use of the development, but a new era in reforestation is beginning.

CARE OF PETS IN THE ELEMENTARY SCIENCE CLASSROOM

GRACE CURRY
Cleveland Museum of Natural History, Cleveland, Ohio

In a recent article we tried to present the educational value of having live pets in the elementary classroom. Even though we are convinced of their value in the science program, the very practical problem of their care arises.

The teacher has two objectives in solving this problem. One is to make the care function as part of the child's educational development, and the other is to make it as easy for herself as possible, so that it will not be just one more burden in an already crowded school day. In addition, of course, the animal's comfort and health should also be given consideration.

It is impossible to give a definite set of rules for keeping pets, as every school has its own specific physical facilities, financial resources, and teacher personnel. The following ideas are suggestions and methods that have worked in some places, but each teacher will have to adapt them to her own situation.

To get some very practical first-hand ideas from a real schoolroom situation, I visited Miss Lucille Richmond, a teacher at Union School in Cleveland. Union School is typical of the school buildings constructed in the early part of the century and is located in the heart of the industrial part of the city. In spite of a lack of modern equipment, Miss Richmond has kept successfully more pets than any other teacher in Cleveland. Her equipment is inexpensive. Many of the cages are rather crude ones made by the children, but for that reason they take real pride in them. The school has the advanatge of having the science room opening off the home room so that the animals have a special place. Miss Richmond enjoys animals herself, and a visitor immediately sees that her enthusiasm is reflected in the children.

I noticed that all the cages were clean and that there was no odor. When I asked Miss Richmond how she organized the children to perform the necessary duties of cleaning cages and feeding the animals, she told me that no child is forced to care for the animals. It is on an entirely voluntary basis, and each committee functions for two weeks. The committee comes in early in the morning in order to finish before the opening of

school. They immediately spread down a newspaper in front of each cage. The refuse from the cage is placed in the paper, rolled up, and put into the waste basket. The cages are washed if necessary and fresh bedding material or paper put in. A brush and dust pan are handy to sweep up any material that may have fallen on the floor. The waste basket is then taken to the basement so that no odor remains in the room. The committee members also prepare the animals' food for the day.

Since the committees are rotated often enough, Miss Richmond never finds the children tiring of their work. There are very few children who do not participate in the care of the animals. The ones who at first do not volunteer soon learn that the others have the real fun with the animals so they soon are doing their share. This well-organized plan of pupil care has relieved the teacher of the actual work and is developing group cooperation and individual responsibility in the children.

Miss Richmond's children have kept skunks, opossums, guinea pigs, white mice, sparrow hawks, doves, parakeets, turtles, snakes, lizards, frogs, toads, salamanders, goldfish, grasshoppers, and crickets. The guinea pig was the most popular with the children because it was so "cuddly."

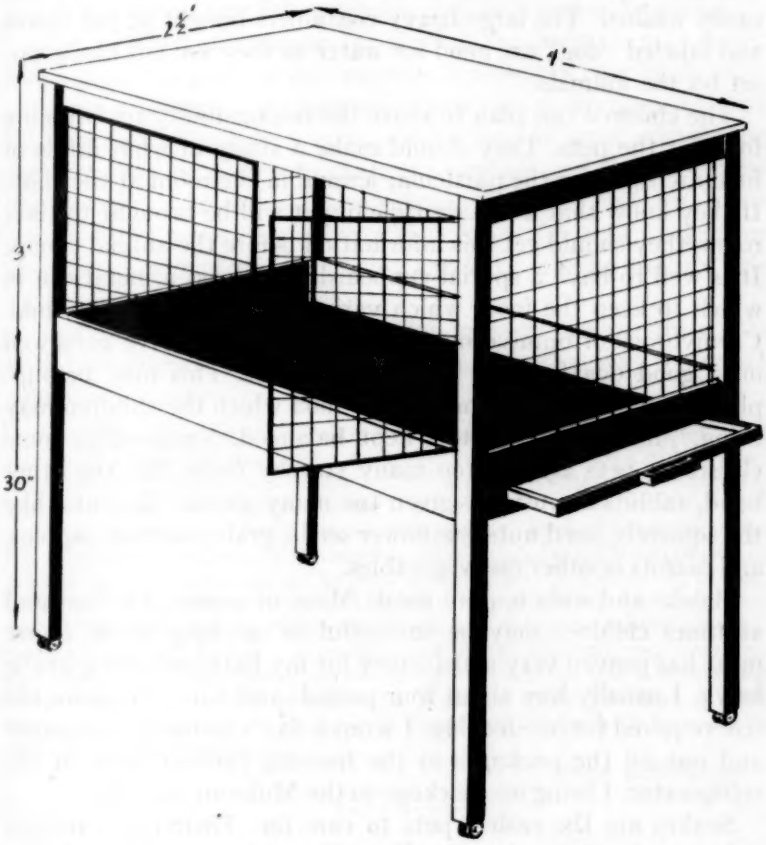
Besides most of the animals Miss Richmond has had, I have also had a baby fox, a raccoon, fox squirrels, flying squirrels, rabbits, a barn owl, and a crow. I found that the children could play with the skunks and the flying squirrels quite safely. But, after the first fear is overcome, snakes become the most popular pets of all. "May I hold the snake?" is the question I have had asked most frequently.

Of course, the first problem with any pet is providing proper housing. The type of cage will depend largely on the ability of the school to finance the equipment. Size is an important factor and depends largely on the animal. A raccoon is active and requires space for movement. Skunks and guinea pigs need less room.

Children can make crude, but usable cages by stretching window screening ,wire mesh, or chicken wire over a frame of the right size. An opening must be provided to make cleaning possible. In some types the top of the cage is a lid which can be raised. In taller cages a door may be built in. A squirrel needs a tree branch to climb on, and birds need perches. In some rooms the cage will stand on the floor and in others it may be on the science shelf. Some times fathers are willing to contribute their

carpentry ability to building a good cage for the classroom. Some schools may find manual training departments willing to build cages.

Since I have to do my own cleaning, I have become very conscious of the problems involved. Some of the Museum staff planned a cage to help solve some of these problems. The accompanying drawing shows the results. This cage may be too expensive for many schools, but it may serve to give ideas which



can be modified to suit individual needs. Some of the advantages are obvious. Since the cage is built on legs, it is not necessary to stoop to clean it or to feed the animals. It is high enough for the children to see the animals from any part of the room. Casters make it easy to move. The removable metal tray facilitates the cleaning. This feature is desirable in any animal cage. Some of

the science supply companies have on sale smaller all-metal cages with a removable tray. These are very satisfactory for the smaller mammals.

For mammals, bedding material should be provided. Baled straw serves the purpose and is not expensive. Rural communities may have access to hay which will cost nothing. Paper on the floor of some cages may be satisfactory for some birds and a few mammals, but many of them will tear the paper in shreds.

Glass or porcelain food containers are good because they are easily washed. The large heavy containers bought at pet stores and labeled "dog" are good for water as they are not easily upset by the animals.

The children can plan to share the responsibility for bringing food for the pets. They should make a study of what kinds of food are right for the particular animal in the room at the time. If they know that a certain type of pet will be brought to their room, they should get this information before the animal comes. It is well to have a special cupboard or at least a metal can in which to keep the food, which varies with the type of animal. Carnivorous or omnivorous mammals seem to thrive very well on a good dog food for their regular diet. This may be supplemented by table scraps or other food which the children may bring, provided the diet is kept balanced. Frequently, some classroom pets are fed too many starchy foods. On the other hand, rabbits are usually given too many greens. Rodents, like the squirrels, need nuts, sunflower seeds, grain, oatmeal, apples, and carrots or other raw vegetables.

Hawks and owls require meat. Mice, of course, are best and at times children may be successful in catching them. Horse meat has proven very satisfactory for my barn owl and sparrow hawk. I usually buy about four pounds and cut it in pieces the size required for one feeding. I wrap a day's ration in wax paper and put all the packages in the freezing compartment of the refrigerator. I bring one package to the Museum each day.

Snakes are the easiest pets to care for. Their cages require only occasional cleaning. Small pebbles make a good foundation for the cage. Snakes should have water at all times, but, being cold-blooded animals, they do not need daily feeding. In the winter they can easily go two or three months without food. Pilot black snakes and others with similar food habits need live mice. These may be purchased at a pet shop or raised for the purpose. Garter snakes prefer toads and frogs, but in the winter

may take meal worms which are bought at pet shops. Water snakes will usually take small fish. Frequently snakes will not eat in captivity and must be force fed. This means that their jaws must be forced apart and meat pushed down their throats. A pencil makes a good "pusher".

Toads do well in a terrarium. Because a frog needs more water than a toad, a terrarium with a pond at one end makes an ideal home. The "pond" is easily made with a shallow pyrex dish sunk in the soil. Horned toads need a desert type of terrarium that is dry and warm. All of these animals may be fed meal worms.

The problem of expense for feeding the animals may arise. Some teachers assume this expense themselves, but if the animals are a part of the educational program it would seem that the expenses should be met in other ways. In some schools the children may be willing to share the expense and at times bring "left-overs" from home. Some school principals may feel justified in using money from the school funds for this project, or the money raised from some school activity may be used for this purpose.

The different classes may develop ways of sharing their pets with the other classes in the building, and in that way the entire school will benefit.

NOTE: The above suggestions are very general and may not answer your specific questions. If you have a question, send it to Grace Curry, Cleveland Museum of Natural History, 2717 Euclid Avenue, Cleveland 15, Ohio. An answer will be forthcoming if possible.

A very useful book for the teacher who needs to have at hand the essential information concerning the housing, feeding, and other items of care of animals in the schoolroom is *The Book of Wild Pets*, by Clifford B. Moore, publisher, G. P. Putnam's Sons. The book is divided into five major sections: The Aquarium, Terraria and Their Small Animal Inhabitants, Insects and Spiders as Pets, Mammals, and Birds. Illustrations, diagrams, and charts add to the usefulness of the book. While the text is written for the layman, the material is authentic. When he wrote *The Book of Wild Pets*, Mr. Moore was the biologist in charge of the trailside museum at Springfield, Massachusetts.

ANNA E. BURGESS

DR. HENRY NORRIS RUSSELL APPOINTED TO HARVARD POST

Dr. Henry Norris Russell, who served as director of the Princeton University Observatory for 35 years before his retirement July 1, has been appointed a research associate at the Harvard College Observatory.

Dr. Russell for many years has been a member of the Harvard Overseers Committee to visit the Harvard Observatory and department of astronomy.

LET'S EXAMINE OUR TESTS

E. B. CHRISMAN
John R. Rogers High School, Spokane, Washington

We would all agree that the student should be given opportunity to learn from his mistakes on a test. But how often is he permitted to review a section of material and take another examination on it? How often is the teacher willing to listen while he explains his interpretation of a question? How many teachers will change a grade if the student shows that he has reasoned beyond the intended confines of the question and arrived at a valid conclusion not anticipated by the questioner? Indeed, how many teachers never permit the student to comment on examination questions at all?

Unvarying, group standardized, treatment of individuals is not desirable in the learning situation in the classroom. Granted that it is proper to recognize individual differences in the learning process in order that the student may make more effective progress, does it not seem sound to provide opportunity for individual expression in the testing program? How can students be expected to conform to the same "testing mold" and still retain their individuality? A student does not lose his unique personality on certain formal occasions at the discretion of the teacher. If grades are to hinge upon examinations, and if the tests themselves are to serve as tools of learning, then students should be permitted the prerogative of interpretation.

What should be the policy, then, when evaluation of educational gain is attempted? There are probably few occasions when use is made of any particular testing technique to the exclusion of others. But how many teachers *consciously try* to provide for variety of expression on tests? Too frequently test booklets are found to use only limited kinds of questions. The true-false, the completion, the matching, the multiple-choice, and the problem-solving types of questions cover most such testing tools. The individual being examined must make the most of his limited opportunities to express himself. Varying possibilities of interpretation of questions is a common test weakness. One result of this is that valuable time is frequently wasted through indecision as to which answer should be given. Examinations as well as the rest of regular class procedure should permit *some* individualized expression.

A few questions of several different types on every major test

can be provided without too much work. In addition to permitting some variety of individual expression, such tests add a touch of color to the process of examination. No one enjoys a steady diet of any one thing. Not knowing exactly what to expect can be stimulating in and of itself. Test booklets that run "true-false," "best-answer," "matching," over and over again, on every test presented, do little but allow an easy road out for the teacher.

What else could be provided on tests? A brief list of some time worn alternatives to the question types mentioned might be provocative of additions:

1. The "set up, but do not solve" type of problem requires all necessary reasoning processes, but does not waste time on mechanical operations, which are often time consuming.
2. The law, or principle stating type of question, requires either rote memory or sufficient understanding to make the statement in the words of the student.
3. The "make a line drawing" type of question can frequently be used quite as effectively as many other types of questions. Simple drawings to be completed by the student can be quite revealing.
4. Brief essay questions, requiring only a sentence or two in answer, are not difficult to check, and allow the individual to qualify his answer if he sees fit.
5. Probably the straight essay type of question is not used sufficiently, either from the standpoint of development of writing ability, or from that of providing for individual expression.
6. Word or number association questions provide variety. In physics for instance, with no suggestive list from which to choose, associations may be required for a list of numbers such as 64; 39.37; 2.2; .433; 62.4; 32.16; 1087; 453.59; 88; 273; 212. Degree of comprehension shows up readily enough in partial answers which the student provides as associations.
7. The "what is wrong" type of question holds possibilities seldom explored by the busy teacher. Equations, drawings, formulas, or statements can be used as a basis for testing analysis and insight.

The purpose of testing should be given frequent consideration. Should we be as concerned as we are with duplication of life situations—i.e., group comparison, and "branding," and

elimination through competition? Or should emphasis be upon course content in relation to our environment, and an attempt to provide well sharpened educational tools? Sight certainly need not be lost of the "whole child" in the examining process. The testing program, being an integral part of educational experience, should be as carefully planned and administered as the rest.

Use made of test results bears constant questioning. Test scores should not be lumped together as a total raw score, and recorded as such in the one and only grade book. If a test is worth the time it takes to construct, it is worthy of being recorded piecemeal. Problems, true false, completion, essay, drawing questions, etc., all should be recorded separately. Record sheets can be provided easily to facilitate score keeping. Obviously enough, remedial direction, counseling, conferences with parents, and final grading, can be much more effective if there is ready access to such diagnostic data.

Tests can be made more interesting than they sometimes are through a little variety of structure and administration. They can be made more meaningful through correlation with immediate environment and experiences. They can be constructed to allow for some individual expression and interpretation on each one. And finally, they can be made more useful through piecemeal recording.

Should we not frequently ask ourselves "Is the school set up as an institution of hurdles, designed to eliminate the weak, or is it a democratic aid to the total growth of each particular individual?" Just because college tests, entrance examinations, and other such qualifying analyses, permit no student recourse, is little reason why the high school should do likewise. Slow students should not be stopped from learning simply because they are slow. Not that everybody should pass all courses, however. Rather that everyone should have all the time available within the semester to learn specified course content. Not all can succeed, even then.

Non-skid mat, designed particularly for home or factory workers who stand long periods at table or bench, has both upper and lower surfaces covered with hundreds of quarter-inch-high rubber cones. It cushions the feet, protects from damp floors and prevents shocks from static electricity.

GEOMETRIC SOLUTIONS OF QUADRATIC EQUATIONS

WALTER H. CARNAHAN
Purdue University, LaFayette, Indiana

It is frequently asserted that Euclid and other ancient Greek mathematicians solved quadratic equations geometrically. This is a statement that no mathematician of Euclid's day would have made, and, made in the twentieth century, it carries a meaning that Euclid and his contemporaries would not have understood.

Quadratic equations, as we understand the expression, belong to algebra, and Greek mathematicians before the time of Diophantus were not algebraists; they were geometers with some interest in number theory. When Euclid spoke of a quantity, the meaning he characteristically had in mind was a line segment. When he talked of the product of two quantities, he usually meant the area of a geometric figure. Generally he drew the lines, rectangles, squares and other figures which his quantities represented. He established many relations among the linear and area quantities, but did not generalize these relations as existing among quantities that are not geometric in the manner that we imply when we write algebraic equations. Before commenting further on this distinction in meanings, let us present Euclid's demonstration of Proposition 11 Book II of *The Elements*. The translation is by T. L. Heath.

PROPOSITION 11

To cut a given line so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.

Let AB be the given line; thus it is required to cut AB so that the rectangle contained by the whole and one of the segments is equal to the square on the remaining segment.

For let the square $ABCD$ be described on AB; let AC be bisected at the point E, and let BE be joined; let CA be drawn through to F, and let EF be made equal to BE; let the square FH be described on AF, and let GH be drawn through to K.

I say that AB has been cut at H so as to make the rectangle contained by AB, BH equal to the square on AH.

For, since the straight line AC has been bisected at E, and FA

is added to it, the rectangle contained by CF, FA together with the square on AE is equal to the square on EF.

But EF is equal to EB; therefore the rectangle CF, FA together with the square on AE is equal to the square on EB. But the squares on BA, AE are equal to the square on EB, for the angle at A is right: therefore the rectangle CF, FA together with the square on AE is equal to the squares BA, AE. Let the square on AE be subtracted from each; therefore the rectangle CF, FA which remains is equal to the square on AB.

Now the rectangle CF, FA is FK, for AF is equal to FG; and the square on AB is AD; therefore FK is equal to AD. Let AK be subtracted from each; therefore FH which remains is equal to HD, and HD is the rectangle AB, BH, for AB is equal

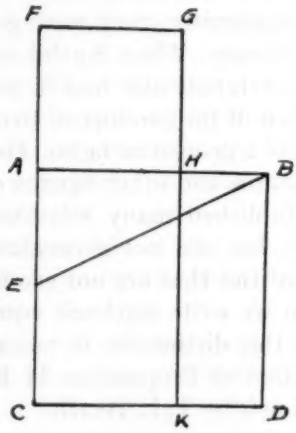


FIG. 1

to BD; and FH is the square on AH; therefore the rectangle contained by AB, BH is equal to the square on HA; therefore the given straight line AB has been cut at H so as to make the rectangle contained by AH, BH equal to the square on HA.

Q. E. F.

Let us represent the length of the given line in Euclid's proposition by a and the length of AH by x. The relations which Euclid has established can now be represented by the equation $a(a-x)=x^2$, but to assert that Euclid here solves a quadratic equation, as Heath does assert, is to attribute to Euclid an achievement which he would never have claimed. In the first place, there was no algebraic equation to be solved, only a geometric condition to be met; and in the second place, the solution

at which Euclid arrives is interpreted to apply only to lines and areas, not to all quantities, as in algebra we interpret them to apply.

In contrast with the geometrically minded Greeks, the mathematicians of old Arabia were algebraists. They approached algebra directly and used geometry merely as a means of clarifying algebraic processes. This statement can be illustrated by the method employed by Alkhowarizmi to solve the equation $x^2+10x=39$. As a means of finding a root of this equation

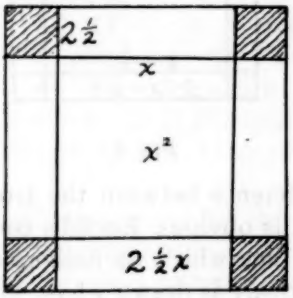


FIG. 2

Alkhowarizmi drew a figure like that here given. A square is drawn having each side equal to x. This is bordered by four rectangles having sides x and $2\frac{1}{2}$ (equals one fourth the coefficient of x in the given equation). In the four corners of this figure are four squares whose combined area is 25 (equals $4\times[2\frac{1}{2}]^2$). The combined areas of the central squares and the four rectangles is 39 as given by the equation. The combined areas of the five squares and the four rectangles is $39+25=64$. Hence the side of the large square is 8. It follows that $x+2\frac{1}{2}+2\frac{1}{2}=8$, or $x+5=8$, and $x=3$.

We may briefly outline the application of this method to the solution of an equation in which the coefficient of the variable is negative. Our illustration is not taken from Alkhowarizmi. Let it be required to find a root of the equation $x^2-x=2$. Draw a square (see Fig. 3) each side of which is x. In this draw lines making rectangles whose sides are x and $\frac{1}{4}$ (equals one fourth the coefficient of x). There are thus formed four corner squares the area of each of which is 1/16. From the figure we see that $x^2-4(\frac{1}{4}x)+4(\frac{1}{4})^2=x^2-x+\frac{1}{4}$ is the area of the central square.

From the equation we have $x^2-x=2$, so $x^2-x+\frac{1}{4}=2\frac{1}{4}$, or $x-\frac{1}{2}=1\frac{1}{2}$, whence $x=2$.

Note that in the above solutions we use only the positive root, as was done by the Arabs. The Greek and Arabic geometric methods are not adapted to finding negative roots.

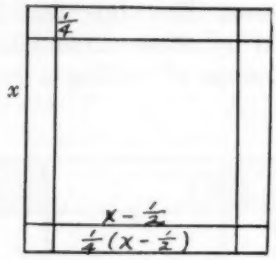


FIG. 3

A very great difference between the Greek and the Arabic geometric solutions is obvious. Euclid's construction gives the exact length of the line which we designated by x, but in the Arabic solution a square is drawn whose side is represented by x but when the drawing is made there is no way of knowing the length of this side; to repeat, the Arabic figure is merely an illustration, and it was not intended to be more than this.

Every student of mathematics soon becomes aware of the very real difference between two expressions, the square *on* x and square *of* x. Euclid, being by nature a geometer, dealt with the square *on* AB or the rectangle *contained by* AB, CD.

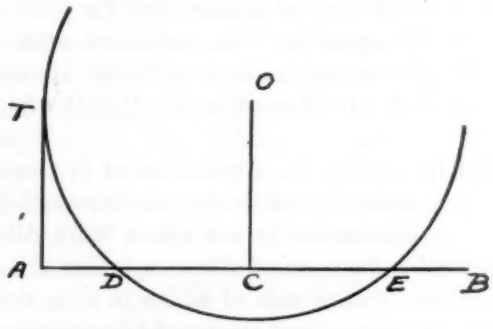


FIG. 4

If we adopt the algebraic rather then the geometric meaning of such quantities as x, a, b, bx, and b^2, the geometric construction

of the root of a quadratic equation is somewhat simplified. The preceding sentence may appear to be a contradiction in terms, but what is intended is to deal with lines and the products of their measures as quantities without showing the squares and rectangles which Euclid would have regarded as essential.

Consider the construction, Fig. 4. Angle TAB and angle ACO are right angles. $AT=CO$ and $AC=CB$. A circle is drawn with center at O tangent to AT and intersecting AB at D and E. It is readily seen that $AD=EB$. Since AT is a tangent and AE is a secant, $AD\times AE=(AT)^2$. Suppose OC is to be a constant whose length is represented by b. (1) If AB is a known length equal to a and AD is an unknown length x, then $x=AD\times AE=(AT)^2$ becomes $x(a-x)=b^2$. (2) If DE is a known length a and AD is an unknown length x, then $AD\times AE=(AT)^2$ becomes $x(a+x)=b^2$. (3) If DE is a known length a and AE is an unknown length x, then $AD\times AE=(AT)^2$ becomes $x(x-a)=b^2$.

Constructions based upon each of the three preceding hypotheses are elementary, being reducible to the construction of a right triangle having two sides given. Consequently the figure gives means for finding geometrically roots of the quadratic equations $ax-x^2=b^2$, $ax+x^2=b^2$ and $x^2-ax=b^2$. Of course b (or a) can be irrational so that b^2 can be any positive quantity.

A simple geometric construction of the roots of a quadratic equation is given by L. E. Dickson. Let $x^2-ax+b=0$ be the

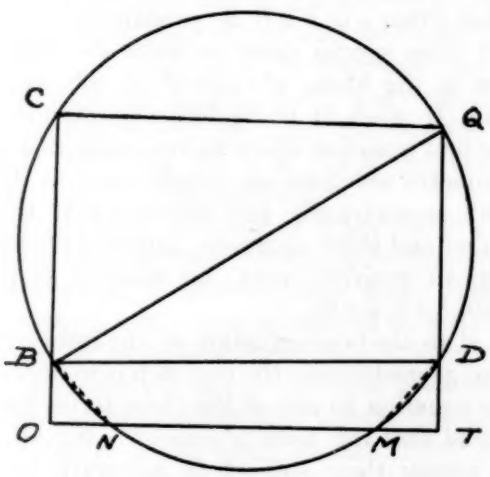


FIG. 5

equation. Construct a rectangle $CBDQ$ having $BD=a$ and $DQ=b-1$ and circumscribe a circle. Extend CB to O and QD to T making $BO=DT=1$. Draw OT intersecting the circle at N and M. Then ON and OM are the roots of the quadratic equation. Dickson's demonstration consists in assuming OC and OT as axes of coordinates and writing the equation of the circle. This equation is

$$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b+1}{2}\right)^2=\left(\frac{a}{2}\right)^2+\left(\frac{b-1}{2}\right)^2.$$

If $y=0$, this equation becomes $x^2-ax+b=0$ which is our original quadratic equation. In other words, the circle intersects the x-axis at points whose abscissas are the roots of the equation. That the abscissas are properly related to the constants of the given equation is thus shown: $OC=b$, $DT=OB=1$, $BN=DM$. Triangle BON is congruent to triangle DTM. $OM+ON=a$, and $OM\times ON=OC\times OB=b\times 1=b$. Since the sum of ON and OM is a and the product is b, therefore OM and ON are the roots of the equation.

It will be observed that besides the lengths a and b Dickson assumes a unit length BO. Of course, Dickson's method of proof of his construction cannot be used in a first or second course in high school algebra because some knowledge of analytic geometry is required to set-up the equation of the circle. This method also makes use of the elementary theory of equations.

The question naturally arises as to modifications of geometric solutions when either a or b is 0. As preparation for a very brief discussion of these special cases we write the three equations solved above in the forms $x^2-ax+b^2=0$, $x^2+ax-b^2=0$ and $x^2-ax-b^2=0$. If $a=0$, it is algebraically obvious that $x=b$ except in the first equation where no real value of x exists if b is real. The geometric solutions are equally obvious. If $b=0$, it is obvious, both geometrically and algebraically, that $x=0$ or $x=a$ in the first and third equations. Limiting ourselves, as did the ancients, to positive roots, no solution is possible for $x^2+ax-b^2=0$, $b=0$, $a>0$.

Of course, if an algebraic equation of the form $rx^2+sx+t=0$ is to be solved geometrically, the first step is to divide by r, thus reducing the equation to one of the three forms solved above. It will be noted that the form $x^2+ax+b^2=0 (a>0,\ b>0)$ does not appear among those subject to geometric solution. The reason is obvious.

THE VOCABULARY OF CHEMISTRY

PART II

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There is one chemical dictionary written by Hachk and one of applied chemistry by Thorpe which may be found in all departmental chemical libraries. Hachk's dictionary presumes to cover the nomenclature in chemistry, physics, astrophysics, minerology, pharmacy, agriculture and biology. There are seven volumes of Thorpe's dictionary, but the pages are written more like an encyclopedia than a dictionary. Words have a habit of changing their meaning and in this way reflect the change of thought and habits of life. This necessitates revision of text books and revision of methods of teaching. It implies that the chemistry teacher needs the technique of the language teacher with an unbelievably large vocabulary at his command. In fact, the problem of teaching vocabulary in chemistry is greater and more difficult than teaching the technique of either mathematics or physics. In organic ehcmistry I have at times assigned the preparation of a list of the new words encountered in each chapter with a definition of their meaning. I have felt that each new chapter in a chemistry text book, like each new chapter in either a French, a German or an Italian grammar, should have a vocabulary or glossary to aid the student in comprehending and interpreting the teaching of the chapter assignment. In another project I prepared the vocabulary for students in general chemistry in each chapter, not only in English, but also in the French and in the German. The hope and expectation was to prevail upon the teachers of those languages to introduce courses in chemical French and chemical German during the freshman year. This idea has been put in practice at Purdue University and in other like institutions.

It may be interesting to inqure as to the size of a vocabulary which a student who majors in chemistry is expected to acquire during his four years of undergraduate study. The answer to this question may receive an approximate answer by a critical study of the index in different textbooks. I have selected five of our texts in general chemistry. In Sneed's *General Chemistry* the index shows approximately 2640 words; In McPherson and Henderson, 3010; in Babor and Lehrman, 3250; in Deming's *Introductory General Chemistry*, 2565; in Schlessinger's, 3200.

The vocabulary of nomenclature in analytical chemistry, i.e. qualitative and quantitative analysis, will approximate 1000.

In organic chemistry the nomenclature averages between 3000 and 4000 words. Gattermann's organic preparations averages about 1000. Physical chemistry will average about 1300. Physiological chemistry, which I regard as a finishing or review course for all the chemistry studied in the undergraduate college, tops the list with over 4000 words.

This array of numbers seems to suggest that a student during his four years of undergraduate study in chemistry alone must grapple with a vocabulary aggregating approximately 10,000 words. I hardly need to make mention of the fact that very many of the scientific terms have meanings which are very involved, such as equilibrium, hydrogen ion concentration, oxidation and reduction, hydrogenation, hydrolysis, electrovalent, covalent and the many theories, living and dead. In addition, if the student has ambitions to continue his studies in the graduate school and takes on the burden of acquiring a reading knowledge of French and German, he will need to acquaint himself with about 4000 technical terms in German and a like number of terms in French. Furthermore, if one takes into account vocabularies which he must acquire in other studies, then must we who teach realize the burden of achievement expected of our youth of this modern age. To learn a vocabulary either in a language or a science requires a vast amount of practice or drill. In the language courses offered by the Government to men in uniform during the last World War it was demonstrated that a language could be acquired in a very short period of time, if a vast amount of time was devoted to practice in both writing and speaking. When we stop to consider the meager portion of time we devote in class to drill on vocabulary and equation writing in our courses in chemistry, it is little wonder that students in particular, and mankind in general, find the field of science a formidable field to conquer. Yet in spite of this difficult situation text books are becoming more voluminous while the preparation of our high school students is becoming less and less adequate for the quests which we expect them to make. Students shrink from all language study. At the University of Florida I find a deplorable lack of interest in language study on the part of students. The lack of high school training in mathematics is also a serious handicap for the teacher of chemistry. In time of war this lack of training threatens to become a national disaster.

In this paper two words have been used, vocabulary and nomenclature. Vocabulary is the general term and includes the number of words used in speaking and writing in any language. Nomenclature is a specific term and applies to the conventions of a science as the Geneva system of naming organic compounds. In teaching general chemistry or organic chemistry it is often found helpful to stress nomenclature by teaching word derivation. Some authors, but not many, have under nomenclature in the index page many references to nomenclature. I find this very helpful in the study and the teaching of the conventions of a science. Out of 8 textbooks examined in general chemistry, only 3 used the term nomenclature in the index. Out of 12, in organic chemistry which I examined, six made no use at all of a nomenclature classification.

I am stressing nomenclature because of its importance in the classification of knowledge in any science, as well as its use in describing natural phenomena. To start with we make a general classification of chemistry into inorganic chemistry and organic chemistry. But the basis for this division is stated differently by both inorganic chemistry and organic chemistry authors. The modern division of chemistry into inorganic and organic is based upon the fact that carbon is the one essential constituent in all organic compounds. But some authors class all carbonates, cyanagin and ammonium cyanate as inorganic compounds. One author states that the great achievement by Wöhler in 1828 was to demonstrate that an organic compound could be synthesized from an inorganic compound. There is always a borderline in any classification and the borderline between inorganic and organic chemistry lies in the compounds CO, CO_2, H_2CO_3. The advantage of this distinction is thus primarily didactic. Science does not have many loose terms but authors are not always careful in the use of chaste chemical phraseology. It is true that the term organic chemistry may be classed as a misnomer due to the fact that in its earliest meaning it was thought to involve a vital force in the formation of compounds of carbon. The term oxygen is also a misnomer, meaning in the original acid former. The German equivalent is "sauerstoff." Carbohydrate is also a misnomer, implying that all compounds containing the constituents hydrogen and oxygen in the ratio of 2:1 are to come under one classification. Fermentation in its original meaning referred to chemical changes resulting in the evolution of a gas. The terms oxidation and reduction are

terms which are very misleading and should be displaced by terms that more nearly suggest the electron theory. This is an outstanding case of using a pattern of classification which is confusing to the mind of the beginner in general chemistry and which belongs to a primitive method of classification. To teach from the beginning the formation of electrovalent or covalent compounds by the transfer or the sharing of electrons would simplify the learning process.

The two terms very badly or incorrectly taught to chemistry students are calorie and percentage. The definition given by students generally for calorie reads, "A calorie is the amount of heat required to raise one gram of water one degree." If you require the student to give you the mathematical concept of percentage you will find that he makes no distinction between percentage, common fractions and decimal fractions. The student will reduce a common fraction to a decimal fraction (or a ratio to a decimal fraction) and state that he has determined a percentage relationship. If we are agreed that one of the important goals of all science teaching is to point out and to learn relationships or make classifications, then we are compelled to teach common proportions. Students and even some college professors have a very vague idea as to the meaning of percentage by volume or percentage by weight. Several years ago I was so annoyed by the many mis-statements from students when I asked them to define "calorie" that I wrote to the Bureau of Standards for the accepted definition. I quote his reply: "The calorie has been defined in a great variety of ways, none of which has up to the present acquired the status of the standard. The calorie was originally defined in terms of the heat capacity of water. Of the numerous possible calories, the 15° calorie (0° to 100°) was much used. Another type of definition is now coming into use. In The International Critical Tables, the equivalent of the 15° calorie is given as 4.185 Joules. This was originally an experimental constant, but it may be considered as the definition of a calorie. In most of the recent thermochemical work done in this country, the calorie has been defined as 4.185 Joules, which is equivalent to 4.1833 International Joules. Another definition of the same type was sponsored by the International Steam Table Conference in 1929. This definition is one calorie equals 1 international watt hour/860."

It is quite generally conceded that classical scholars assigned names to many of our chemical elements. As was pointed out

earlier in this paper, few students who elect chemistry as a major have had adequate training in language other than the American language. As to training in Greek or Latin, the number is almost nil. Therefore, when you attempt to teach a student with no language background that Na is the symbol for sodium, Ag the symbol for silver, Au for gold, K for potassium, Pb for lead, you find you have to impose the task of pure memorizing. To add to this confusion you expose the student to the names of multitudinous compounds derived from the Greek and the Latin. It is incumbent upon the teacher of chemistry to spend a great deal of time in teaching word derivation. After he has done his utmost he is conscious of the fact that he has wasted his effort hopelessly. In most cases because he has no background on which to build. The only thing that I see that has been done to remedy this situation is to slaughter the innocents by the thousands by issuing "flunks."

In this paper I have stated a problem which is common to all teachers and authors but I have not produced a solution. I have found nothing in current literature dealing with this specific problem or its solution. Probably all teachers in all fields are grappling with the same situation.

WORLD'S FIRST COMMERCIAL PRODUCTION OF SYNTHETIC VITAMIN A ANNOUNCED

The world's first commercial production of synthetic vitamin A was announced here today by Distillation Products, Inc., of Rochester, N. Y.

The announcement was made in a paper presented by Dr. James G. Baxter before the American Chemical Society's fall meeting in the Hotel Pennsylvania.

"This discovery should help bring vitamin A in high purity within the the reach of all who need it," R. W. Albright, the company's general manager, said in an explanatory statement to the press.

Vitamin A is essential for proper growth and vision. Hitherto the only commercial source of vitamin A has been the oil extracted from the livers of such fish as the cod, dogfish, and soupfin shark.

"The supply of vitamin A from fish is highly uncertain but the new product does not have such a limitation," Albright pointed out. "Vitamin A was the first vitamin to be discovered but it is the last major vitamin to be synthesized."

In explaining the new product, Dr. K. C. D. Hickman, director of research of the company, emphasized that the successful synthesis resulted from teamwork by the group of six researchers.

"Production of the synthetic vitamin A is now proceeding in a 'pilot plant' and samples soon will be available for test purposes. The company does not wish at present to discuss how the synthesis is done or what materials are used," Albright said. The product will carry the trade mark "Myvax."

THE CALIBRATION OF LABORATORY INSTRUMENTS

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There are quite a number of instruments used in a wide variety of scientific laboratories that require calibration before they can be put to use in analytical work. By "calibration" is meant the correlation of instrument readings with the units of the quantity the instrument is supposed to be measuring. More specifically, this might mean the correlation of thermocouple millivolts with degrees Centigrade, photofluorometer readings with vitamin units, colorimeter readings with grams sugar, or standard colored tubes correlated with known ph (hydrogen ion) concentrations for the purpose of measuring acidity.

The basic mathematical ideas underlying these cases of calibration involve the use of known standards, the plotting of the instrument readings against the known units of the standards, and the construction of a graph or curve between the known points, mathematically known as interpolation. The standards are reagents of known purity obtainable at chemical supply houses.

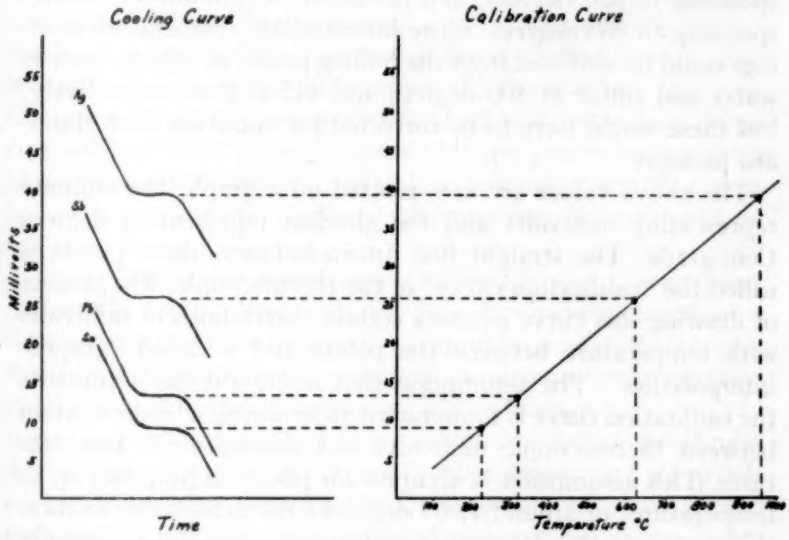
It is the belief of the writer that this general procedure can be taught and illustrated on the level of secondary science and mathematics. Following is a description of one such calibration suitable for this purpose. The materials may be obtained fairly cheaply from apparatus companies.

CALIBRATION OF A THERMOCOUPLE

A thermocouple is an instrument used in metallurgical laboratories to measure temperatures beyond the range of the ordinary mercurial thermometer. It consists essentially of two dissimilar wires, such as the alloys alumel and chromel, twisted together at one end and connected to a millivoltmeter or potentiometer by copper wire from the other two ends. (One voltage instrument must be used consistently before and after calibration, since the millivoltmeter readings are somewhat less than those of the potentiometer because of the internal resistance of the circuit.) When the twisted end is placed in a furnace, a tiny electrical current is produced, whose electromotive force (EMF), or millivoltage, is measured by the voltage instrument.

This voltage is called the Seebeck electromotive force.[1] Some high school students may be interested in reading about the theory of the thermocouple, but this is not essential in order to understand the calibration of the instrument.

For a chromel-alumel thermocouple, the millivolts produced range from about 4 at 100°C. to 52 at 1,300°C. with increments of roughly 4 millivolts for each 100 degrees of rise. Although these figures are accurately known for a theoretically perfect thermocouple, they are far from reliable on a new thermocouple, chiefly because of impurities in the thermocouple and because



the junctions of the thermocouple with the copper wire leads are not held at 0°C. Each new thermocouple must therefore be separately calibrated so that its millivolts may be accurately converted to temperature readings.

Several pure, crystalline substances are used, such as tin, lead, antimony, and silver. Their freezing (or melting) points are known to be as follows: tin (Sn) 232 degrees centigrade, lead (Pb) 327 degrees, antimony (Sb) 630 degrees, and silver (Ag) 960 degrees. It is taught in high school physics that whenever a pure crystalline substance, such as water or tin, melts or freezes, its temperature remains constant at its melting point until all of

[1] For a theoretical discussion of the Seebeck EMF, see Lemon, Harvey Brace and Ference, Michael, Jr., *Analytical Experimental Physics*, Chicago, The University of Chicago Press, 1943, pp. 336–341.

the substance has changed state due to the latent heat of fusion. This principle is applied here. The first substance, tin, is placed in a crucible and heated in a furnace until molten. The thermocouple is placed in the molten tin, which is then allowed to cool outside the furnace. Millivolt readings are recorded every minute. When the millivolt readings remain constant for several minutes, the freezing point has been reached. For tin, the millivolt reading at this point would be approximately 9.5, corresponding to a temperature of 232 degrees Centigrade. For lead, the reading would be 13.5 millivolts, corresponding to 327 degrees; for antimony, the reading would be 26 millivolts, corresponding to 630 degrees, and for silver, 39.6 millivolts, corresponding to 960 degrees. Some intermediate temperature readings could be obtained from the boiling points of liquids, such as water and sulfur at 100 degrees and 445 degrees respectively, but these would have to be corrected for variation from standard pressure.

The above values are now plotted on a graph, the ordinate representing millivolts and the abscissa representing degrees Centigrade. The straight line drawn between these points is called the "calibration curve" of the thermocouple. The process of drawing this curve assumes certain correlations of millivolts with temperature between the points and is called "graphic interpolation." The assumption that a straight line represents the calibration curve is tantamount to assuming a linear relation between thermocouple millivolts and thermocouple temperatures. This assumption is accurate for practical purposes up to temperatures of around 1,000 degrees Centigrade. The calibration curve of other laboratory instruments may not necessarily be linear.

When accurately drawn with India ink on labeled graph paper, the calibration curve may be framed and mounted near the millivoltmeter or potentiometer for reference whenever the thermocouple is used. The accompanying graphs will show how the calibration curve is constructed from the cooling curves. The actual calibration curve should have longer scales than those shown in order to give more refined readings. (The slopes of the cooling curves will vary according to the specific properties of the substance used.)

Finally, education alone can conduct us to that enjoyment which is, at once best in quality and infinite in quantity.—HORACE MANN.

THE ORIGINS OF OUR NUMERAL NOTATION*

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Sigfried Giedion in the foreword of *Space, Time and Architecture* says, "History is not a compilation of facts but an insight into a moving process of life." I know of no better illustration of this statement than the history of the origins of our numeral notation.

The numeral notation now employed by the peoples of Western Europe and by the peoples stemming from them came into widespread use in the seventeenth century, the century of Descartes, Napier, Stevin, Newton, Leibniz. It has four characteristics: (1) it is a decimal system; (2) the numbers from one through nine are represented by symbols which stand for things counted and stand for nothing else; (3) the principle of position or location which makes it possible to use only these nine symbols with the zero to represent any number; (4) a symbol for nothing, zero.

Most histories of mathematics, especially the shorter ones, discuss the origin of our notation in a cursory way mentioning that so far as its introduction in Western Europe in the twelfth, thirteenth and fourteenth centuries is concerned, the notation came from the Arabic speaking peoples who in turn had obtained it from India; hence it is usually denoted as the Hindu-Arabic notation. Carl B. Boyer in the summary of an article on "Fundamental Steps in the Development of Numeration" believes that "our system might better be known as the Babyl nian-Egyptian-Greek-Hindu-Arabic" system. Just to make the picture complete so as to include the forms of the numbers I would add after Greek the name Roman.

The system is a decimal system. Except for the sexagesimal system[1] of the Babylonians, who also used a decimal system, a decimal system was adopted by civilized peoples in all parts of the world. Hence one is forced to conclude that its origin is related to a fundamental characteristic of man, for each normal member of the genus homo sapiens has ten fingers and ten toes.

* A paper read at the December meeting of the Mathematical Club of Tulane University.

[1] In the sexagesimal system the numbers above the units which extend from one to fifty-nine are arranged according to the powers of sixty. Sexagesimal fractions were used by the Greeks in astronomical calculations. The circumference of the circle was divided into 360 parts. This system appears fully developed in Ptolemy's "Almagest." Its origin may be accounted for in prehistoric man's estimation of the number of days in a year as 360.

It would be a mistake, however, to assume that because of its universal adoption in ancient times and by the peoples of today that it is therefore the most logical and convenient system, an assumption often made today by those who would extend the decimal system to the measurement of length, mass, etc., in the English speaking world. It may be the most logical, but there are those who argue that a system based on the number twelve would be more convenient since twelve is divisible by three and four.

The principle of position or local value was used by the Babylonians and Egyptians who also employed the repetitive and additive principles. The number 324 was written by the Babylonians and Egyptians with three symbols for 100 followed on the right by two symbols for 10 and four unit marks. The classical civilizations of Greece and Rome developed systems based on Egyptian hieroglyphics, introducing however some slight modifications. Like the Babylonians the Greeks and Romans used a special sign for five. The method of repeating signs became basic. The Romans used also the subtractive principle, writing IX instead of VIIII. Large numbers written in the Attic system and in the Roman system were neither easy to write nor to read.

The Ionian Greeks, centered around the city of Miletus, devised the most satisfactory system of numeration of the ancient world. It is my opinion that the Ionian alphabetic system was the foundation stone of the system developed by the Hindus sometime after the second century A.D. So far as I know there is no direct evidence for this opinion, and because of the difficulties encountered in fixing within a century or two any advance in Hindu mathematics or science there most probably never will be any conclusive evidence brought forward either for the idea or against it. The origin of the Hindu system will probably never be known with certainty. The Ionian system, later called the Alexandrian alphabetic notation, used signs for numbers, the signs being based on the Greek letters of the alphabet used in Ionia before the fifth century B.C. The Greeks took their alphabet from the Phoenicians, but they did not get this idea of using letters to represent numbers from the Phoenicians, who like the Egyptians had separate signs for numbers. The Phoenician alphabet had twenty-two letters, to which the Greeks added some, the vowels. In devising their numerals, the Ionians used twenty-seven letters, three groups of nine each:

the first nine stood for the units; the next nine for the first nine integral multiples of ten; the last nine for the first nine integral multiples of one hundred. The earliest date showing the use of this system in an inscription is about 450 B.C.

The fundamental idea of the Ionian system did not originate with the Greeks. Their notation was the result of the development of a principle made use of by the Egyptians in their hieratic or demotic numeration[2] in which easily written signs are used for the units, for integral multiples of ten, one hundred, one thousand, ten thousand. This principle rendered unnecessary the primitive device of iteration characteristic of other notations. The Ionian system was simple and easily learned and used, since the signs were already known as the letters of the alphabet. Paul Tannery in one of his memoirs published in 1882 states that he practised himself in the use of the Ionian system and found that he could carry out calculations in it almost as rapidly as he could do them in our system.

It is to be noted that in the alphabetic notation the principle of position is not necessary but the Greeks wrote the letters in the order which had been adopted in the older systems, placing the letter for the largest number to the extreme left, the others following in the order of their value.

The spread of the Ionian notation over the Greek world of the Mediterranean Basin was slow. It was officially adopted at Alexandria by Ptolemy II, Philadelphus. The letter numerals were used most advantageously in inscriptions and on coins when the custom arose of writing on coins and monuments the year of the reign of the ruler at the time. The Ionian alphabetic numerals have been found on coins which have been assigned the date 266 B.C. The period immediately following the adoption of this notation was the golden age of Greek mathematics, the age of Archimedes and Apollonius. The alphabetic numerals were introduced in Athens sometime before 50 B.C. and by 50 A.D. they were in official use.

The numerals from one through nine which we use today are derived from the so-called ghubār (also written ghobār and gobar) numerals. Three kinds of ghubār numerals are recognized: those used by the western Arabs in Spain in the last half of the middle ages from which modern numerals are derived; those used by mathematicians writing in Arabic who worked in or

[2] In the mathematical papyri, such as the Rhind, the hieratic notation is used. Hieroglyphics were reserved for formal inscriptions.

near Bagdad in the ninth century; those found in Latin manuscripts written in the latter part of the tenth century. The numerals used by the eastern Arabs now and since about the beginning of the twelfth century must have been derived from certain Hindu numerals in use in the period from 900 to 1100 A.D. They were script numerals employed by peoples trading with the Arabs and with the peoples of India, most probably the Persians. Authorities seem to differ somewhat as to the exact forms of these early Arabic numerals. However a comparison of them with our numerals indicates at once that they are not like modern numerals. For example, the five looks like our zero and the zero is written as a dot. On the other hand all the modifications of the ghubār numerals show certain features in common and resemble modern numerals.

The ghubār symbols for 2, 3, 4, 5 are similar to the numerals of the Hindu Nâgarî script. These forms for 2, 3, 4, 5 are found in Buddhist manuscripts and in inscriptions which are dated from 500 to 800 A.D. This period is antecedent in time to the period when the Arabic speaking peoples began to use the Hindu numeral system. Moreover, there is no zero in these numerals. The forms of the Nâgarî numerals are believed to have been derived from the Brâhmî script numerals which were in use in India probably from about 1000 B.C. Our symbols for 6 and 7 are like those of the Brâhmî script found in a cave in the Bombay area, which writing has been assigned as of the second century A.D.

So far as we know today the oldest European manuscript containing the ghubār numerals is a Latin manuscript written in 976 A.D., the so-called Boetius geometry. The author shows in his work that he knows nothing of the Hindu system of calculation with the numerals. The symbol for zero is missing. The numerals are associated with the abacus which was used extensively by the Romans for purposes of calculation.

All evidence that I know of seems to point to the development of the ghubār numerals as they are written in the Latin manuscripts of the tenth century and as they are written by the Arabic peoples in Spain from Hindu numerals brought to Spain during the time it was a part of the Roman Empire. There is a possibility that the numerals may have arrived in Spain via Alexandria, Egypt. They were probably introduced by traders from India or by adventurous native Spanish traders who carried on the business of exchanging merchandise with the coastal cities of India and the islands near it. In a recent number of the

National Geographic Magazine, Vol. XC, November, 1946 there is a statement that in the excavation of a fortress in central Ceylon, the historical monument of Sigiriya, 1687 coins were found of which all but twelve were Roman or Indo-Roman bronze, dating from A.D. 317 to 423.

Alexander von Humboldt in the second of those most interesting volumes entitled, *Cosmos* tells us that the Greek geographer Strabo of the Roman Empire who lived in the first part of the first century A.D. remarked in his Geography that while in Egypt he noted vessels sailing from the port of Nyoshormos on the Red Sea to India. Humboldt also says that in the time of Marcus Aurelius (A.D. 121–180) the Romans traded and exchanged legates with the rulers of China and India. There would seem to be plenty of evidence to show that traders in merchandise from India might easily have brought the Hindu numerals used at that time in the coastal regions of India to the shore towns of western Europe. It is highly probable that Boetius (A.D. 480–524) knew of them and wrote of their use with an abacus in a work that has been lost.

The earliest Arabic exponent of the Hindu arithmetic with the use of the numerals, the principle of position and the zero was al-Kwarīzmī who wrote in the first quarter of the ninth century (c. 825 A.D.). He is known by the name of his birth place, Kwarizm, modern Khiva, a town south of the Aral Sea, now in Russian territory. His influence on the development of mathematical thought is believed to be greater than that of any other medieval writer. The word algorist, one who uses Hindu numerals, is derived from his name. The best exposition of the Hindu system was written by al-Bīrūnī, a Persian encyclopaedist of the eleventh century.

The Hindu system was introduced into the Latin world by Adelard of Bath, an Englishman, who became an algorist and who probably translated al-Kwarīzmī's mathematical treatise from Arabic into Latin sometime between 1130 and 1142. In 1202, Leonardo of Pisa, more often called Fibonacci, published a book, *Liber Abaci*, which gave a complete and systematic explanation of Hindu numerals. Fibonacci had lived on the Barbary coast where his father was a commercial agent of Pisa and there he had learned Arabic and the methods of calculating with Hindu numerals. In using the Arabic word zifr (empty) he changed it into the word zephyro, whence our word zero. Fibonacci was an excellent mathematician.

The use of the Hindu numeral system spread slowly. It was

not needed in business since only results were recorded and the calculations could be quickly and accurately done by means of an abacus; there was no social need for the system for the age of admiration for statistics was not to begin for several centuries. They are not found in the celebrated Alfonsine Tables prepared in the latter part of the thirteenth century. It is believed however that some practical men of northern Italy used them in this century. It might be of interest to mention that in Spain during the last part of the medieval period, where both Arabic and Latin were written, compilers used the Hindu numerals when writing in Arabic, but used the Roman numerals when writing in Latin.

The important features of the Hindu system are the principle of position and the use of the zero. These are great inventions. Were they original with the Hindus? Very probably not. There are records which show that the Alexandrians in writing a number like 3223 often wrote it $\gamma\varepsilon\kappa\gamma$ which clearly indicates that the position of a numeral signified its value in terms of integral multiples of ten. Moreover the Hindus were not the first to use a small circle as the symbol for zero. Ptolemy used the letter *0* to indicate a blank position in his sexagesimal system. The use of a special symbol for a blank position in a number goes back to the Babylonians who needed such a symbol in their astronomical calculations. If the principle of position and the use of a dot or small circle for the zero were independently thought out by Hindu mathematicians sometime following the beginning of the second century B.C., as is claimed by Datta and Singh in their *History of Hindu Mathematics* (p. 87), then it must be admitted that these mathematicians constitute the least observant and most narrowly circumscribed intellectual group of the ancient civilized world, for in the six centuries following the beginning of the third century B.C. they must have come in contact over and over again with the Ionian-Alexandrian system of notation. Datta and Singh state that the earliest use of the principle of position in a mathematical work occurs in a manuscript written about 200 A.D. Even with the Hindus whose attitude toward time is so different from ours four hundred years seems a long time between the appearance of a mathematical idea expressed in a sacred poem and its appearance in a work on mathematics. If one accepts the date of the origin of the principle of position in India as about 200 B.C. and also accepts the statement of Datta and Singh that it was only after 500 A.D.

that the use of the principle of position with the zero became widespread, it becomes evident that the development of the Hindu system extended over a period of seven hundred years. Furthermore it might be of interest to note that the century following 500 A.D. marks the peak of the development of Hindu mathematics.

So far as we know the first important contact of India with Greek civilization was made when Alexander the Great in his career of conquest, having traversed ancient Parthia, entered what is now northern Afghanistan, crossed the mountains, then crossed the Indus River and followed the river to the sea. When Alexander died in 323 B.C. his empire was divided among three of his generals. Seleucus became the ruler of the eastern portion of the empire. The successors of Seleucus, the Seleucids, the capital of whose empire was Antioch in Syria, never lost sight of one objective, namely, to found cities like those of the Mediterranean basin and to spread Hellenism among the conquered peoples. The peak of the military and political power of the Seleucids was attained in the second century B.C. at which time Menander's Indian empire reached its greatest size. The second century B.C. marks the close of the period of the highest development of Greek mathematics; it is the period immediately following the work of Apollonius who came to Alexandria from Pergamum in Asia Minor; it is the period when the Ionian numeration came into official use in Alexandria, the coins of the realm being stamped according to this system. Alexandria was not only the greatest center of learning in the world, but what is more important to this story, it was a thriving, cosmopolitan, commercial city with communication by sea with India and Ceylon, for at this time a canal connected the Pelusiac branch of the Nile River with what is today called the Bitter Lakes which empty into the Red Sea.

It is probable that the principle of position and the idea of a symbol for an unfilled place in a number were taken over by the Hindus from the Ionian system and from the calculations of Babylonian-Alexandrian astronomy with both of which they were in contact from the third century B.C. to after 200 A.D.

What then is the contribution of the Hindu mathematicians? It is the logical development of these ideas into a system of calculation which is easy to understand, easy to learn and use; a system which makes the writing and reading of any number an operation of only a few seconds. This was a very, very great

contribution not only to the development of arithmetic but also to the development of all sciences based on measurement. It is a contribution which the mighty Greeks of the third or second century B.C. should have made but did not.

Summary

1. From its many origins our numeral notation should be known as the Babylonian-Egyptian-Greek-Hindu-Arabic-Roman system.

2. The symbols for modern numerals are like the ghubār numerals which were derived from certain unknown Hindu numerals without the zero, and which were introduced into western Europe, probably Spain, about the second century A.D.

3. The Hindu numeral system involving the principle of position and the use of the zero, as it was passed on to the Arabic speaking peoples in the eight century A.D., was developed in India after 200 A.D.

4. The great contribution of Hindu mathematicians was the development of a system of calculation involving only nine number symbols, the zero and the principle of position. The ideas of the principle of position and the zero were taken over from Ionian-Alexandrian mathematics with which Hindu mathematicians had been in contact for at least four hundred years before 200 A.D.

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Without ideals, without effort, without scholarship, without philosophical continuity, there is no such thing as education.—Abraham Flexner.

METHODS OF SOLVING ELEMENTARY SYSTEMS OF EQUATIONS IN TWO UNKNOWNS

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Elementary systems in two unknowns are solved in beginning algebra courses by presenting special devices. These devices are unsatisfactory because (1) they lack completeness in failing to show altogether a simpler system that is inconsistent, (2) the resulting systems may not be in their simplest forms, (3) confusion may arise in the grouping of equations to form new systems, (4) no reason for a device may be given.

By considering two equations such as $x+y-3=0$ and $2x-3y+4=0$ the concept of a linear combination can be introduced. Let $u\equiv x+y-3$ and $v\equiv 2x-3y+4$. Then $u+kv$, where k is an arbitrary constant, is a linear combination of u and v. The equation $u+kv=0$ will be satisfied by every pair of values that satisfies $u=0$ and $v=0$. And every pair of values that satisfies $u=0$ and $u+kv=0$ must satisfy $v=0$. Or, every pair of values that satisfies $v=0$ and $u+kv=0$ must satisfy $u=0$. Hence the system

$$u=0$$
$$v=0$$

may be replaced by

$$u=0$$
$$u+kv=0$$

or, by the system

$$v=0$$
$$u+kv=0.$$

And this holds for equations of higher degree.

The graph of $u+kv=0$ goes through all the points of intersection of $u=0$ and $v=0$ and cannot meet either of them in any other point. The line $(x+y-3+(2x-3y+4)=0$ is shown in Fig. 1 going through the intersection of $x+y-3=0$ and $2x-3y+4=0$, k having been given the value 1.

A typical example of the method used in algebra texts[1] for solving the system

$$x^3+y^3=999 \tag{1}$$

$$x+y=3 \tag{2}$$

is the following: Divide (1) by (2),

$$x^2-xy+y^2=333. \tag{3}$$

Square (2),

$$x^2+2xy+y^2=9. \tag{4}$$

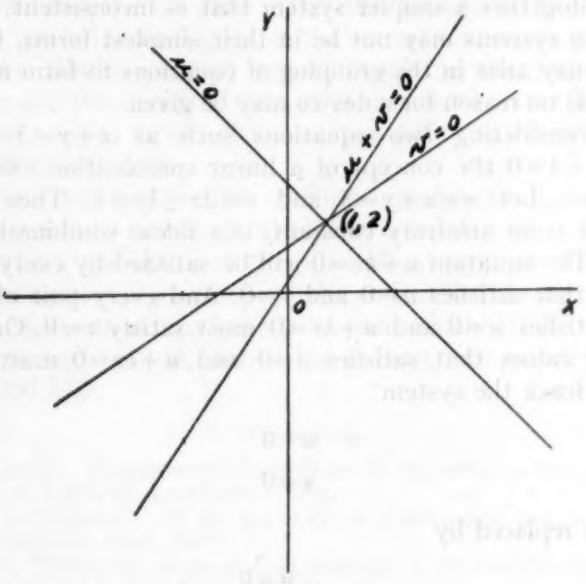

FIG. 1

Subtract (4) from (3),

$$\begin{aligned} -3xy&=324 \\ -xy&=108. \end{aligned} \tag{5}$$

Add (3) and (5),

$$x^2-2xy+y^2=441. \tag{6}$$

Take the square root of (6),

$$x-y=\pm 21. \tag{7}$$

[1] *College Algebra*, Rouse.

Combining (2) and (7) there result the two cases

$$x+y=3 \qquad x+y=3$$
$$x-y=21 \qquad x-y=-21.$$

Use of the linear combination

$$(x^3+y^3-999)-333(x+y-3)=0,$$

which can be written

$$(x+y)\,[x^2-xy+y^2-333]=0$$

has the advantage of showing that the resulting additional system

$$x+y=3$$
$$x+y=0$$

is inconsistent.

As another example[2] the system

$$x^2+3xy=28 \tag{1}$$
$$xy+4y^2=8 \tag{2}$$

is solved by the special device of "eliminating the constant."

Multiply (1) by 2: $2x^2+6xy=56.$ (3)

Multiply (2) by 7: $7xy+28y^2=56$ (4)

Subtract, (3)−(4): $2x^2-xy-28y^2=0;$ or,

$$(2x+7y)(x-4y)=0. \tag{5}$$

To solve (1) (2), (2) (5) may now be solved. This system is equivalent to the following simpler systems:

$$\begin{array}{ll} xy+4y^2=8, & xy+4y^2=8, \\ 2x+7y=0. & x-4y=0. \end{array} \tag{6}$$

And it is stated that instead of using (2) in (6), (1) could equally well have been used. Use of linear combinations is not restrictive, however. Thus,

$$2(x^2+3xy-28)-7(xy+4y^2-8)=0$$

and

$$(x^2+3xy-28)+(xy+4y^2-8)=0,$$

[2] *College Mathematics*, Hart, Wilson and Tracy.

give the simpler systems:

$$\begin{cases} x+2y-6=0 \\ 2x+7y=0 \end{cases}$$

$$\begin{cases} x+2y-6=0 \\ x-4y=0 \end{cases}$$

$$\begin{cases} x+2y+6=0 \\ 2x+7y=0 \end{cases}$$

$$\begin{cases} x+2y+6=0 \\ x-4y=0. \end{cases}$$

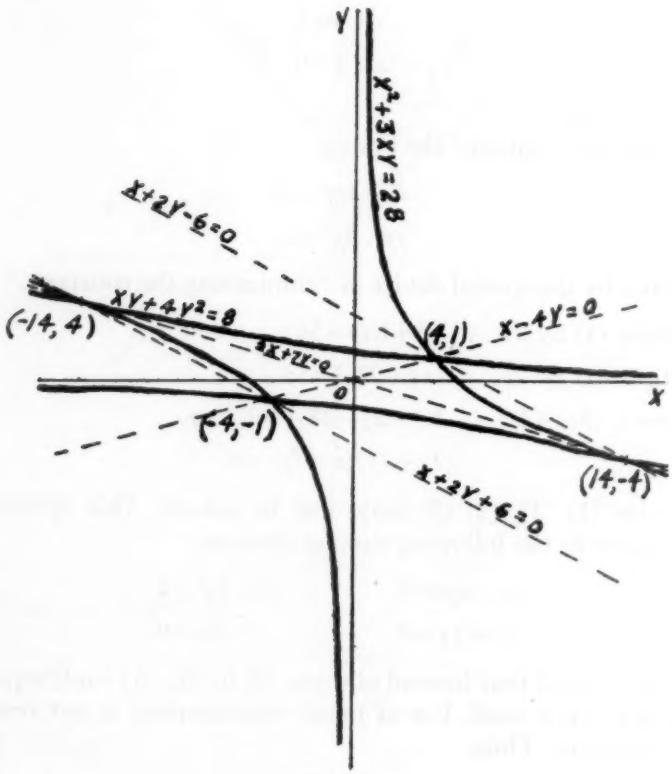

FIG. 2

The confusion in grouping equations to form equivalent systems is eliminated when linear combinations are used, for graphical illustrations can be given. Thus, in the above example, one could see that it would be redundant to solve (5) with both

(1) and with (2) since (5) must go through all the intersections of (1) and (2) and meet them in no points which are not intersections (see Fig. 2).

When all terms involving the variables are of the second degree a system such as

$$x^2-xy+y^2=3 \tag{1}$$

$$x^2+2xy=5 \tag{2}$$

is solved[3] by letting

$$x=wy. \tag{3}$$

Substituting in (1) and (2)

$$w^2y^2-wy^2+y^2=3 \tag{4}$$

$$w^2y^2+2wy^2=5. \tag{5}$$

Solving (4) and (5) for y^2,

$$y^2=\frac{3}{w^2-w+1} \tag{6}$$

$$y^2=\frac{5}{w^2+2w}. \tag{7}$$

Equating the values of y^2 given by (6) and (7),

$$\frac{5}{w^2+2w}=\frac{3}{w^2-w+1}.$$

Clearing of fractions, there results

$$2w^2-11w+5=0 \tag{8}$$

which has the roots 5, and $\frac{1}{2}$. Substituting 5 for w in (6).

$$y^2=\frac{1}{7}.$$

Hence

$$y=\frac{1}{\sqrt{7}} \quad \text{or} \quad -\frac{1}{\sqrt{7}}.$$

Since $x=wy$, the value of x to go with

$$y=\frac{1}{\sqrt{7}} \quad \text{is} \quad x=\frac{5}{\sqrt{7}}.$$

[3] *College Algebra*, Ford.

Similarly, when $y=-1/\sqrt{7}$, $x=-5/\sqrt{7}$.
Likewise the solutions (1, 2) and $(-1, -2)$ are obtained.

On the other hand, since all the terms involving the variables are of the second degree, a linear combination can always be found which is reducible and gives the multipliers, w, at once. Thus,

$$5(x^2-xy+y^2-3)-3(x^2+2xy-5)=0$$

is reducible by the quadratic formula to $x=5y$ and $x=\frac{1}{2}y$. As explained above, these lines go through the intersections of the curves of equations (1) and (2) and meet the curves in no other points.

THE QUIZ SECTION

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1. Two drops of mercury coalesce to form a larger drop. Is there a change of temperature?
2. How does the period of a pendulum change if the pendulum is lengthened by 1 percent?
3. A horizontal board is moved up and down with simple harmonic motion of period 0.5 second. What is the maximum amplitude possible in order that a weight placed on the board may not be jerked off?
4. If this same board (#3) were moved *horizontally* with simple harmonic motion, travelling through a distance of 30 inches and making 15 trips (complete oscillations) per minute, find the least coefficient of friction in order that the weight may not slip.
5. If the earth's mass be taken as 6×10^{21} tons, and we assume 10^{10} tons of snow and ice to melt at the North Pole and to move to latitude 45°, how much change would there be in the length of the day?
6. A spool of thread lies on the table. By pulling on the thread properly the spool can be moved either *toward* or *away from* the "puller." Explain.
7. Consider two *identical* iron rods. One is magnetized, the other is not. How can you determine which is which *without any apparatus whatsoever*?

NOTE TO READERS: A number of readers have been good enough to give me their opinions on this QUIZ SECTION. In general, it appears that the problem-query type is preferred. The Editor would be happy to receive other comments.

The following submitted solutions to the problems in the April issue: Hugo Brandt, Chicago; Roy Hatch, East Northfield, Mass.; Robert Brooks, Chicago, F. L. Maxwell, Greer, South Carolina; Robert Shaw, New York.

Prismatic street lamp puts 40% more light on the roadway than earlier types, it is claimed. Because of the many prisms which comprise the glass globe, the light is directed in two broad beams up and down the street.

BLOOD TYPING AND BLOOD INHERITANCE USING STUDENTS AND PARENTS AS SUBJECTS FOR DEMONSTRATION

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During the fall and spring terms of 1946–47 the authors performed a series of tests to determine the blood types of students and parents. These experiments were performed at the Barringer Evening High School, Newark, N. J. In the spring term of 1948 they will be repeated for the General Science students attending the Newark Colleges of Rutgers University. The results achieved convince the authors that a new experiment of vital and contemporary interest can be added to the list suitable for high school and college biology classes.

A. APPLICATIONS

There are several situations for which this simple but timely demonstration is pertinent. Primarily it is meant for high school and college biology students. Since, however, it illustrates the principles involved in the determination of non-paternity suits, this demonstration is also suitable for pre-law students in the social science classes.

B. OBJECTIVES

The authors believe that a number of purposes can be served by this series of experiments. The following list will suggest the widespread interest as well as the educational motivation that may be realized.

1. To observe blood cells in the suspended and in the clumped state.
2. To learn the blood types of several students and their parents, thereby stimulating interest in the physiology of the human body.
3. To evoke parental interest in school activities.
4. To show that blood type is inherited according to certain well founded principles. This objective can be emphasized in the unit on inheritance.
5. To show that blood transfusions cannot be performed in-

discriminately but that successful transfusions depend upon a knowledge of blood grouping.

6. To teach a lesson in racial tolerance, i.e., that all races of mankind have the same blood types and the same blood chemistry.

7. To suggest to college students the principle that underlies the performance of blood tests in suits involving disputed paternity.

C. Underlying Principles of Blood Group Classification

According to Landsteiner,[1] there are four distinct blood

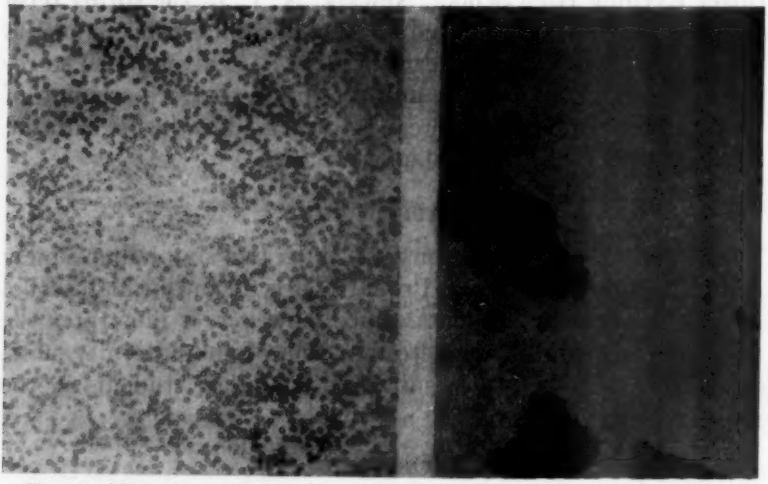


Fig. 1. Negative reaction (no agglutination) on left. Positive reaction (showing intense agglutination of the red cells) on right. From Alexander S. Wiener's *Blood Groups and Blood Transfusions*. Courtesy Charles C Thomas, Publisher, Springfield, Illinois.

groups among the people of the world. These he designated A, B, AB, and O. The determination of these groups depends upon the presence of certain chemical substances both in the red blood cells and in the plasma (de-fibrinated plasma is called serum). The presence of these two chemical groups, the one in the red blood cells and the other within the bathing plasma, may cause clumping of the cells when whole blood from a person in one blood group is mixed with the plasma of a person in another group. See fig. 1. The occurrence of such clumping or aggluti-

[1] Karl Landsteiner, an Austrian scientist, was awarded the Nobel Prize in 1930 for his research in blood grouping.

nation is actually the basis for world blood group classification and for blood typing, a preliminary process used by technicians to determine appropriate donors for a blood transfusion. The significant facts are these.

Among other components, red blood cells contain chemical substances called antigens. The particular antigens which command our attention are the agglutinogens. These substances, never isolated and, therefore, without chemical formula, are nevertheless identifiable by their reaction with specific sera. Depending upon the presence or absence of two particular agglutinogens, A and B, in the red blood cells, Landsteiner established four blood groups among members of the human race, —A, B, AB, and O. The O group has red cells lacking in agglutinogens A and B. The incidence of these four groups in the world population is shown in table 1.

TABLE 1. BLOOD GROUP FREQUENCIES

Group	Prevalence in World Population
A	42%
B	10%
AB	3%
O	45%

The plasma or liquid portion of the blood holds other chemical substances, the antibodies. Among these is a group designated as the agglutinins. Reactions between the agglutinins in the plasma and the agglutinogens in the red blood corpuscles may cause clumping of the cells when blood from two persons is mixed. This event is called agglutination.

According to Landsteiner's concept, a person's plasma holds agglutinins for which there is no corresponding agglutinogen in the red blood cells. For example, the plasma of a group B individual contains only agglutinin a (Anti-A); the plasma of a group A individual holds agglutinin b (Anti-B). Study of table 2 shows the four red blood cell groups according to their agglutinogen content with their corresponding plasma agglutinin content. We must note that *small letters stand for plasma content and that capital letters designate the blood cell content and, therefore, the blood group.*

Now, when whole blood donated by an individual in group A is mixed with the plasma of a group B recipient, the donor's

red blood cells clump. Why? Simply because the plasma of our group B recipient contains agglutinin a (Anti-A) which clumps the red cells of the group A donor with their content of agglutinogen A. Conversely, a donor's blood cells in the group B

TABLE 2. BLOOD COMPONENTS

Cells May Contain Antigen or Agglutinogen	Serum May Contain Antibody or Agglutinin
A	b
B	a
A and B	none
O	a and b

category are agglutinated when mixed with the plasma of a recipient who belongs in the group A bracket. Here, obviously, we have a "conflict" between agglutinogen content B in the donor's cells and agglutinin content b (Anti-B) in the group A recipient's plasma. Clumping of the donor's corpuscles follows.

A logical question now arises. Why don't the recipient's corpuscles likewise clump. Naturally, if equal volumes of blood were mixed there would be a mutual clumping of blood cells. But in transfusions we must consider the dilution factor. For the volume of the donor's blood plasma is so small compared with the volume of the recipient's blood, and the dilution of donor agglutinins is so effective, that the extent of clumping suffered by the recipient's blood cells is negligible. In other words, *the donor's blood cells and the recipient's plasma are "important."* The reverse consideration is unimportant because of

TABLE 3. BLOOD TYPING

Cells	Serum of Group			
	O	A	B	AB
	Containing Agglutinin			
	a and b	b	a	–
O	–	–	–	–
A	+	–	+	–
B	+	+	–	–
AB	+	+	+	–

+ denotes agglutination
– denotes absence of agglutination.

dilution of the donor's blood. In the matter of blood transfusions, it's the possible clumping of the donor's cells and not the recipient's that must command our attention.

Table 3 shows all the possible reactions between the four types of red blood cells, O, A, B, and AB, and the four types of plasma, ab, b, a, and o. An examination of this table as well as an appreciation of the dilution factor make intelligible the 'possible transfusions' shown in table 4. We can readily see that O is a universal donor, and that AB is a universal recipient.

TABLE 4. POSSIBLE TRANSFUSIONS

Donor Group	Possible Recipients	Donor's Blood Agglutinates with
A	A, AB	B, O
B	B, AB	A, O
AB	AB	A, B, O
O	A, B, O, AB	nothing

D. CLASSROOM TECHNIQUE FOR DETERMINING STUDENT'S BLOOD GROUP

The technique is very simple. All that is needed besides a microscope and several glass slides are a needle, a wax pencil, test tubes, a glass rod, 2 pipettes, grain alcohol, saline solution, and samples of serum agglutinin a and b. Serum is de-fibrinated plasma. The a and b sera can be obtained from any local hospital since these specimens must be on hand in order to perform transfusions. Hospital authorities are usually glad to co-operate with schools and colleges in these matters.

Serum samples are to be used immediately, or if they are to be used the following day the samples should be stored in a refrigerator to prevent decomposition by microorganisms.

Specimens of blood are obtained by first swabbing the student's finger with alcohol and then jabbing sharply and quickly with a sterilized needle. Two or three drops of blood may be squeezed out, and these should be transferred to a test tube containing a few cc. of a .9% salt solution.[2] This is done by having the student place his finger over the mouth of the test tube and then inverting quickly. A cherry-colored suspension of red blood cells results. See fig. 1.

[2] To make up the saline solution: to 0.9 grams NaCl, add distilled water to a total volume of 100 cc. Make up fresh or store in the refrigerator.

With a wax pencil, the left end of a glass slide should be marked "A" serum (agglutinin b), the right end of the slide, "B" serum (agglutinin a). With pipettes, one drop of each serum sample is now placed at the appropriate end of the slide, and one drop of the unknown cell suspension is mixed with each of these sera. A simple means of adding the cells is by transferring from the test tube with a glass rod. Serum and cells are stirred. Then the rod, wiped with clean, dry rag, may be used for the next transfer. The slide is then tilted back and forth for a few minutes.

Although this technique differs from the standard method of blood grouping where cells are "washed" before being grouped, it is still adequate for our purposes. This eliminates the need of a centrifuge.

If the two drops are examined microscopically after 5–10 minutes, the blood group to which the student belongs may readily be determined. (The student should also examine the specimens, since this gives him an appreciation for the morphology and physiology of blood.) Consult table 3; look at fig. 2. If neither drop shows clumping, then the blood belongs to group O. If the drop on the left, in agglutinin b, shows clumped cells, but there is none in agglutinin a, then the blood is in group B. Should the cells in agglutinin "a" clump, but not those in b, then the unknown blood belongs in group A. If, however, there is clumping in both sera, then the student's specimen is in group AB.

E. Theory of Blood Group Inheritance

Specific blood groups, like other physical characteristics, are transmitted from generation to generation by means of theoretical entities, the genes, lying on chromosomes within the germ plasm. According to Bernstein's[3] theory, now widely accepted, there are three allelomorphic genes which determine the four blood groups. The two genes for blood group lie on the same loci, in an identical pair of chromosomes, in every somatic cell, for all peoples in the world's population.

One of the three allelomorphic genes determines either the A, B, or O group. See fig. 3. Bernstein says that when parental genes meet in the zygote, A is dominant over O, and B likewise dominates O. The A and B genes, however, are equipotential,

[3] The exact mechanism of heredity was not determined until 1925 when Bernstein solved this problem.

with neither one dominant or recessive. We can, therefore, have only four phenotypic blood groups, A, B, AB, or O, where the genotype is OO for the last mentioned group. Table 5 makes these relationships clear. Here we see that a group A phenotype may actually be either an AA genotype if homozygous, or an AO genotype if heterozygous; the group B phenotype may be either a BB genotype or a BO genotype.

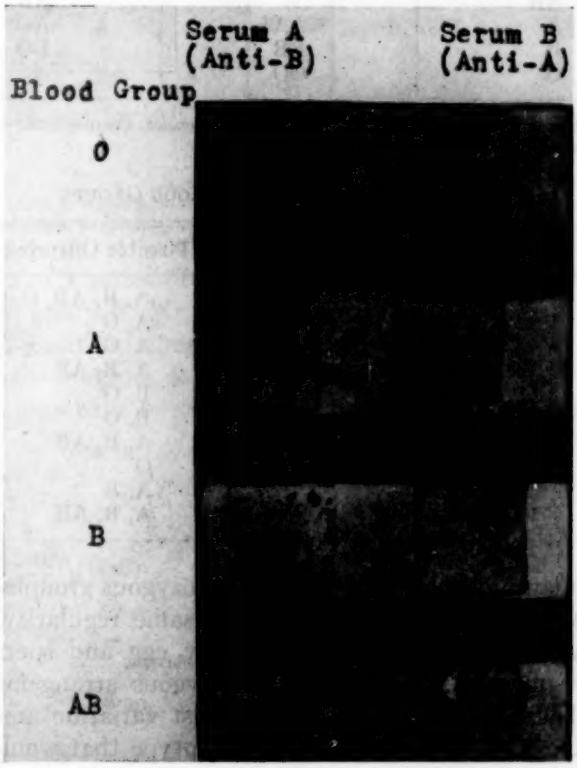


FIG. 2. Blood grouping on glass slides. From Alexander S. Wiener's *Blood Groups and Blood Transfusions*. Courtesy Charles C Thomas, Publisher, Springfield, Illinois.

As to the blood groups that children may inherit through their parent's germ plasm, the possibilities are shown in table 6. Now, by examining the two Mendelian crosses represented in fig. 4, the "possible offspring" shown opposite combinations 1 and 7 in table 6 become intelligible.

Assuming heterozygosity for the blood group phenotypes A,

B, AB, and O, all the other Mendelian crosses can be similarly worked out for each combination shown in table 6.

TABLE 5. BERNSTEIN'S THEORY

Phenotype	Genotype	
	Homozygous	Heterozygous
AB		AB
A	AA	AO
B	BB	BO
O	OO	

From Alexander S. Wiener's *Blood Groups and Blood Transfusion*. Courtesy Charles C Thomas, Publisher, Springfield, Illinois.

TABLE 6. HEREDITY OF THE BLOOD GROUPS

Parents	Possible Offspring
1. A×B	A, B, AB, O
2. A×O	A, O
3. A×A	A O
4. A×AB	A, B, AB
5. B×O	B, O
6. B×B	B, O
7. B×AB	A, B, AB
8. O×O	O
9. O×AB	A, B
10. AB×AB	A, B, AB

In human physiology, of course, homozygous grouping of the two allelomorphs will occur with the same regularity as the heterozygous arrangement in the early egg and sperm. The literature merely suggests the heterozygous arrangement "on paper" because this permits the greatest variation among offspring and naturally includes the phenotype that would result from the homozygous condition.

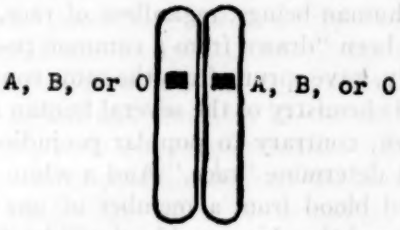
F. CLASSROOM TECHNIQUE FOR DETERMINING INHERITANCE OF BLOOD GROUPS

The student is provided with two small test tubes each holding a few cc. of the .9% salt solution. One tube is labeled "Mother" and the other, "Father." Using the same technique described above, a suspension is made of each parent's blood.

After the student returns the specimens to the classroom, agglutination tests can be run for the two parents and for their

student offspring. On determining the three blood groups, the instructor can consult table 6 and then make the appropriate Mendelian cross on the blackboard to demonstrate how a blood group is inherited by offspring from parents.

FIG. 3. Mechanism of Heredity by Triple Allelomorphs



From Alexander S. Wiener's *Blood Groups and Blood Transfusion*. Courtesy Charles C Thomas, Publisher, Springfield, Illinois.

In one of a series of tests run by the authors, Mrs. K. belonged to group B, Mr. K. to group O, and the daughter Jane, to group B. In another test, Mrs. R. was in group A, Mr. R group AB, daughter Rose, group A. From the data on hand, Mrs. K. may have been either a BO or BB genotype; similarly,

FIG. 4. Illustrations of Law of Segregation.

(1) A×B

Assuming heterozygosity,

P_1	AO	×	BO	
Gametes	A	O	B	O
Possible F_1 Genotypes	AB	AO	BO	OO
Phenotypes	AB	A	B	O

(2) B×AB

Assuming heterozygosity,

P_1	BO	×	AB	
Gametes	B	O	A	B
Possible F_1 Genotypes	AB	BB	AO	BO
Phenotypes	AB	B	A	B

Mrs. R. may have been either an AO or AA genotype. In the case of each maternal parent in these two tests, either the homozygous or heterozygous arrangement of genes could have produced the blood group observed in the offspring.

G. OBJECTIVES REALIZED

The first five objectives, mentioned above, can be realized

from the context of the paper. The last two objectives, on reflection, must be obvious to a biologist. Certainly, the objective dealing with racial tolerance should be brought to the attention of students. As Dunn and Dobzhansky of Columbia University have pointed out, the fact that the four blood groups occur in the same approximate proportion among all races of mankind indicates that human beings, regardless of race, color, and nationality, have been "drawn from a common pool." All races of men, apparently, have sprung from the same common ancestors. Since the blood chemistry of the several human races seemingly is identical then, contrary to popular prejudice, "blood" certainly does not determine "race." And a white person who refuses transfused blood from a member of one of the colored races on the ground that his own blood will be "tainted" has no scientific basis whatever for his thinking.

The last objective, that dealing with the use of agglutination tests in cases of disputed paternity, can be grasped by studying table 6. Obviously, such tests are negative in their character. They can only rule out an alleged parent; they cannot prove paternity.

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The authors are grateful to the Newark Beth Israel Hospital Laboratories for their generous contributions of sera and to the Charles C Thomas Publishers for permission to reproduce figures 1 and 2.

TEN EVENTFUL YEARS

"Ten Eventful Years," the Encyclopaedia Britannica's definitive new history of mankind's most significant decade—the years 1937 through 1946—was published in August.

The new four-volume work is a detailed history of the last ten years of business, science, politics, culture, world affairs, the war and almost every other important human activity. It is unique in that it is written largely by the very persons most important in the events described, and was completed while these events were still fresh in the contributors' minds.

Eight hundred of the most prominent persons in the world comprise the select list of authors of this new four-volume work, which includes 1,500 articles totaling 3,100,000 words, and is illustrated by 1,300 photographs, charts and maps.

Among the contributors are 23 heads or former heads of countries; authorities representing 40 different nationalities; a cross-section of the top Allied military leadership in World War II; scientists and sociologists: philosophers, artists, authors and business executives.

SLIDE RULE INSTRUCTION FOR STUDENTS OF HIGH SCHOOL CHEMISTRY

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Nicholas Senn High School, Chicago, Illinois

During the war years there was an increased emphasis placed upon mathematics and its applications in the field of science. As a result the trend toward fewer problems in high school chemistry was halted and reversed. In the writer's classes the lack of proficiency with simple arithmetical operations severely limited the amount of problem solving which could be done in the time available for that phase of chemistry. To remedy this situation the use of the slide rule was encouraged for the simple divisions and multiplications needed in elementary chemical calculations.

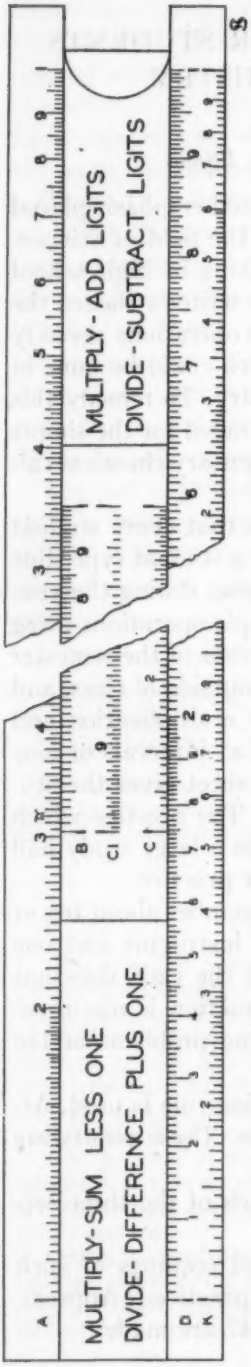
The student response was so enthusiastic that every student in the writer's classes is now required to buy a student type slide rule and to use it for all of the problem solving during the year of chemistry. Instruction is given in the simple operations using the *C* and *D* scales. During the first three weeks of the semester several series of four lessons each are given outside of class, and each student must attend one series. Only multiplication and division are taught. The lessons are spaced at intervals of four or five days, but a mimeographed practice sheet gives the student the opportunity to use his rule daily. The prestige which the student gains by using his "slip-stick" in a large study hall has in this school served as an incentive for practice.

The slide rule instruction groups usually number about ten or fifteen students. They are handled by the instructor and one student assistant. They meet on days that the class does not have a double laboratory period. The instruction is non-theoretical and is planned specifically for solving problems of the types encountered in high school chemistry.

For the first lesson a six foot demonstration rule is used. Attention is given only to the *C* and *D* scales. These points are covered:

(a) The values of the sub-divisions in each of the three sections of the rule are noted.

(b) With the slide removed, settings and readings of such numbers as 1.27, 2.24, and 7.75 are practiced. Approximate settings for numbers such as 5.47 are made.



(c) The handling of such numbers as 273 and .025 is explained.

(d) With 1 on the *C* scale above 2 on the *D* scale, the hair-line is set on various numbers on the *C* scale and the hair-line number on the *D* scale is read.

(e) Simple multiplications are made using small integers, the decimal point being disregarded or estimated.

The second lesson is largely instruction in fixing the decimal point in multiplication. Estimation requires more 'mathematical sense' that is possessed by a great number of the girls (and boys) who elect a year of chemistry. Former students who had learned slide rule operation in certain army schools brought back the method which seemed the most teachable. Determining the number of digit places to the left of the decimal point is the basis of the method. Since extensive use is made of this number, it seemed advisable to give it a short name. The term "ligit" was suggested and adopted. A ligit is defined as a digit place to the left of the decimal point. Thus 26.575 has two ligits and 3.65 has one ligit, while .75 has zero ligits and .054 has negative one ligit. Negative ligits were first pictured by putting a -1 inside each zero of such a number as .(−1)(−1) 54 to show its negative two ligits.

In simple multiplications, the number of ligits in the product is equal to the sum of the ligits in the two factors, UNLESS the slide extends to the right, in which case it is the sum less one. The pupil is required to write down the ligit calculation and when the extension to the right occurs, a small (*R*) is marked on the problem. Completed exercises appear in this form:

	Problem		*Ligit calculation*
(1)	$8\times25=2$	becomes 200.	$(1+2\rightarrow3)$
(2)	$.08\times2.5=2$	becomes .2	$(-1\ +1\rightarrow0)$
(3)	$.4\times.2\ (R)=8$	becomes .08	$(0+0\ -1\rightarrow-1)$
(4)	$.003\times.03\ (R)=9$	becomes .00009	$(-2\ -1\ -1\rightarrow-4)$

Practice sheets with numerous two factor multiplications were required and the determinations of the decimal points were written as above.

The objectives of lesson three were the mechanics of simple division and the fixing of the decimal point in the quotient. When the facts involved in multiplication are firmly fixed, this lesson is the easiest of the series. Division on the slide rule might well be taught before multiplication, but in the writer's opinion the fixing of the decimal point in the product should be learned before that in the quotient.

In division, the number of ligits in the quotient is the number of ligits in the dividend less the number of ligits in the divisor, UNLESS the slide extends to the right in which case it is the difference plus one.

Usually the algebra of subtracting negative numbers must be reviewed at this point. But after the first day the student will write the ligit computation for

$$\frac{4.8}{.003\ (R)} \quad \text{as} \quad (1\ +2\ +1\rightarrow4) \quad \text{rather than} \quad 1-(-2)+1\rightarrow4$$

In several cases students have used a toy electric wood-burning pencil to mark their slide rules as in the figure below, in which the position of the slide exposes the proper instructions. This scheme may have some value at first, but with a little practice the ligit calculation becomes as mechanical as the reading of the slide rule.

Since most of the problems in elementary chemistry involve a combination of a single division and a single multiplication, the fourth and last lesson considers such problems. In such a combination as $\frac{3}{4}^2\times9$ the student is asked to perform the division first and then the multiplication, reading the answer only after the final hairline setting.

The fixing of the decimal point in the answer is somewhat

simplified by the fact that the extension of the slide to the right or the left can be ignored as long as the slide extends in the same direction for both operations as it does in the large majority of cases.

Problems such as $\frac{8}{4}\times 2$ are not written $\frac{8}{4}\times 2\ (R)$ since the (R)'s cancel and can be disregarded. If, however, the indices must be changed during the computation, the (R) must be indicated in the division or the multiplication where it occurred. For the following problems the ligit calculations are as indicated.

$\frac{25}{5}\times 3 \quad (2\ -1\ +1\rightarrow 2) \qquad \frac{25}{5}\times 12\ (R) \quad (2\ -1\ +2\ -1\rightarrow 2)$

and but

$\frac{54}{18}\times 12 \quad (2\ -2\ +2\rightarrow 2) \qquad \frac{54}{18\ (R)}\times 9 \quad (2\ -2\ +1\ +1\rightarrow 2)$

By the time the student has completed the instruction outlined above, his chemistry lectures and his laboratory experiments are introducing problems, and he feels that the time spent in learning the slide rule operations has not been wasted. Problems are much more interesting to him when mathematical drudgery is eliminated and practice with this new tool is substituted.

The instructor feels that it has been worthwhile also. Instead of requiring the student to think through and work out completely one or two problems in a single class period, he now can assign several problems with confidence that they can be completed in the same length of time. Another satisfaction comes when former students returning from universities and technical shops report that the ability to perform simple operations on the slide rule, is one of the skills which they have found most useful since leaving high school.

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LOUIS AGASSIZ, TEACHER OF SCIENCE

JAMES D. TELLER
The Adjutant General's Office, Department of the Army

Louis Agassiz was one of the great science teachers of all time, and one of the most famous. He was famous because he was able to captivate all manner of men, scientists and poets, philosophers and mechanics, emperors and farmers. William James felt that not since Benjamin Franklin had we had among us a person of more popularly impressive type. Jules Marcou thought that his popularity even rivalled that of the great statesman, Daniel Webster. His broad humanity enabled him to awaken an enthusiasm for science among all classes of people whether rich or poor. A consideration of the sympathetic, human side of his character and his genial, personal influence should enable us to better understand the secret of his great success as a teacher.

1. *Agassiz and his students*

Agassiz especially delighted in the love and approbation of his students. And he secured their admiration, not by producing a school of disciples, but by the sheer force of his personal power. It is true that the influence of his personality sometimes gave his opinions a weight beyond the value of the investigations on which they were based. Thus, John Fiske, on reading the headline "The Darwinian Theory utterly demolished by Agassiz Himself," asked, "Is there a scientific pope among us?" Nevertheless, Agassiz attempted to teach his students that they should not follow him but should learn to think for themselves. That his teaching was effective is evidenced by the fact that "every one especially trained by him afterward joined the ranks of the evolutionists." As illustrative of the experience of his students we may cite that of David Starr Jordan. He at first accepted Agassiz's philosophy regarding the origin and permanence of species. However, he gradually found it impossible to believe that the different kinds of animals and plants were separately created in their present forms. As Jordan sometimes said "I went over to the evolutionists with the grace of a cat the boy 'leads' by its tail across the carpet!" Yet Jordan appraised Agassiz as "the greatest teacher I have known."

Jordan felt that all of Agassiz's students went through an experience similar to his own. Perhaps it was such experiences of

his own students that caused Agassiz to confess to one of them in the last year of his life:

> The greatest mistake of my scientific life has been in fighting the theory of evolution. I saw that it was coming for years and my "Essay on Classification" was written largely to forestall it. I believed it all wrong, but now I see that it will prevail.[1]

We have chosen Jordan to represent that exceptional group of students whom Agassiz had at Penikese. Of the classes at the Lawrence Scientific School, Morison rated the first to receive the S.B. degree, the Class of 1851, as "the most distinguished." This class of four members was composed of William L. Jones, later of the University of Georgia; Joseph LeConte later professor of biology in the University of California; John D. Runkle, later president of the Massachusetts Institute of Technology; and David A. Wells, later a noted economist. We shall let LeConte speak for this group.

> I respond the more willingly to your invitation to say something in honour of Professor Agassiz, because I owe personally so deep a debt of gratitude to him as my teacher. For some fifteen months in the years 1850-51, as his private pupil, I spent the whole working hours of nearly every day by his side, either in the laboratory or else in excursions along the shores of Massachusetts, or over the mountains of New York, or on the reefs and keys of Florida. The result of this long intimate association was, on my part, a great and ever-increasing love, admiration, and reverence for him, both as a scientist and as a man, and on his part, I am sure, a very strong and affectionate regard."[2]

Nathaniel Southgate Shaler, distinguished between Agassiz the instructor and Agassiz the educator, and then evaluated each as follows: "Agassiz was the worst instructor I have ever known, but in diverse ways the greatest educator." It was in that memorable Autumn in which Darwin's *Origin of Species* appeared that Shaler first met Agassiz. In recalling this meeting Shaler spoke of it as "my first contact with the man who was to have the most influence on my life of any of the teachers to whom I am indebted." Again, it was Agassiz's personal quality that captivated young Shaler.

Among Shaler's fellow-students in the Lawrence Scientific School were Samuel H. Scudder, Frederic W. Putnam, Addison E. Verrill, Alpheus Hyatt, and William James. Of this group none wrote more extensively about their master than did the last named. Shortly after enrolling in the school to study chemistry under Charles W. Eliot, who had just been placed in charge of the chemical laboratory, James writes to his parents:

> Agassiz gives now a course of lectures in Boston, to which I have been. He is evidently a great favorite with his audience and feels so himself. But

he is an admirable earnest lecturer, clear as day, and his accent is most fascinating. I should like to study under him.[3]

Two months later he feels able to place the desired study with Agassiz in proper perspective with the rest of his program. He proposed this outline:

. . . one year study chemistry; then spend one term at home, then one year with Wyman, then a medical education, then five or six years with Agassiz, then probably death, death, death with inflation and plethora of knowledge.[4]

Of course not all of this youthful prophecy came to pass immediately for thirty-five years later he recalls one of the great benefits which he derived from his studies with Agassiz:

We cannot all escape from being abstractionists. I myself, for instance, have never been able to escape; but the hours I spent with Agassiz so taught me the difference between all possible abstractionists and all livers in the light of the world's concrete fulness, that I have never been able to forget it. Both kinds of minds have their place in the infinite design, but there can be no question as to which kind lies nearer to the divine type of thinking.[5]

Speaking at the Agassiz centennial at Cornell University on May 28, 1907, another of Agassiz's students, Burt G. Wilder, gives still other reasons for his gratitude to his old teacher:

I am one of the few survivors of those who were directly associated with Agassiz as pupils, assistants, or colleagues. He inspired me with interest, with admiration, with respect, nay, almost veneration. No shadow ever came between us. Whatever benefits he may have conferred upon others, I have reason to believe that, outside his family circle, there is no one, living or dead, who has such cause for gratitude and affection in return for counsel, for encouragement, for opportunity, and even for material aid in the form of specimens or information.[6]

In addition to the four students whose appreciations we have quoted, many others have expressed their gratitude to Agassiz. Various lists of Agassiz's more prominent students have been prepared. *The American Naturalist*, founded by four of Agassiz's pupils in 1867, took occasion on the golden anniversary of Agassiz's first class at Harvard to publish a list of thirty-nine names, and concluded that "when we look over the names of those who are doing the zoological work of America today, we find few who have not been trained by Agassiz, his pupils, or by his pupils' pupils." In 1928 Warren published a list of sixty-eight names "of friends or students" secured by searching the early reports of the Museum of Comparative Zoology and the memories of two surviving students of Agassiz. The complete "Catalogue of the Anderson School for 1873" is contained in the

Report of the trustees for that year. The eminence of the names in these lists serve to show Agassiz's ability not only to single men out but also to inspire them to investigate the problems which were pressing for solution. Moreover, since the majority of them became teachers of biology, they have spread Agassiz's influence as a teacher over the globe.

2. *Agassiz's scientific friends*

Agassiz's scientific friends included some of the greatest thinkers of the nineteenth century. With some of these he formed life-long friendships in which the influence was reciprocal. The influence of Alexander von Humboldt on him is well known, but it is not unlikely that Agassiz also exerted an influence on Humboldt. Although an opponent of Agassiz's glacial views, he treated him tenderly for such a sharp and sarcastic critic. Thus, he writes to Agassiz on August 15, 1840:

> . . . I cannot close this letter without asking your pardon for some expressions, too sharp, perhaps, in my former letters, about your vast geological conceptions. The very exaggeration of my expressions must have shown you how little weight I attached to my objections. . . . My desire is always to listen and to learn.[7]

Almost two years later, in a letter expressing his continued incredulity to Agassiz's glacial views, Humboldt concludes:

> . . . But I am only a grumbling rebellious subject in your kingdom. . . . Do not be vexed with a friend who is more than ever impressed with your services to geology, your philosophical views of nature, your profound knowledge of organized beings. . . .[8]

Although Agassiz failed to convert his Prussian friend to his glacial theory, he early converted his English friend, Dr. William Buckland, Dean of Westminster, to his point of view. It was Dr. Buckland who acted as his guide in the hunt after fossil fishes and in the search after glacier tracks in Great Britain. These journeys mark the beginnings of life-long friendships with Adam Sedgwick, Charles Darwin, Sir. Charles Lyell, and Sir Roderick Murchison. All of these except the latter almost from the beginning accepted Agassiz's glacial views, and even he came to accept them before the end of his life.

Sedgwick met the "famous foreign fishmonger," as he once referred to Agassiz, at a meeting of the British Association for the Advancement of Science in Edinburgh in 1834. The following year Agassiz visited Sedgwick at Cambridge and arranged to draw the fossil fishes which Sedgwick had had brought together

for him. These two friends were not only glacialists but also creationists.

In a letter dated August 19, 1868, Darwin thanked Agassiz for information which he had supplied on the Amazonian fishes and added that he had never for a moment doubted his kindness and generosity. He recalled that ever since they first met at a meeting of the British Association for the Advancement of Science that he had felt for Agassiz "the warmest admiration." While Darwin accepted Agassiz's glacial views, Agassiz never accepted Darwin's evolutionary views. However, Darwin with his characteristic perspicacity had prophesized in the *Origin* that it would be impossible to convert the older naturalists to his theories. Yet each recognized the intellectual integrity and greatness of the other. Thus, when Henry Wadsworth Longfellow was traveling in England in 1868, he wrote to Agassiz of the delight with which he found his memory so beloved in England:

> In the Isle of Wight, Darwin said, "What a set of men you have in Cambridge! Both our universities put together cannot furnish the like. Why, there is Agassiz,—he counts for three."[9]

Lyell accepted both the glacial views of Agassiz and the evolutionary views of Darwin. Yet he remained the friend of both. It was he who was instrumental in making the arrangements which secured for Agassiz his first course of Lowell Lectures.

Agassiz helped Sir Roderick Murchison in naming and describing certain fossils for his *Silurian System*, "one of the landmarks in the history of geology." However, for many years Murchison remained an opponent of Agassiz's glacial theory. Thus, in a letter to Sedgwick describing the meeting of the British Association for the Advancement of Science in 1840, he writes:

> Agassiz gave us a great field-day on Glaciers, and I think we shall end in having a compromise between himself and us of the floating icebergs! I spoke against the general applications of his theory.[10]

Twenty-two years later, with rare candor, Murchison writes to Agassiz as follows:

> I send you my last anniversary address, which I wrote entirely myself; and I beg you to believe that in the part of it that refers to the glacial period, and to Europe as it was geographically, I have had the sincerest pleasure in avowing that I was wrong in opposing as I did your grand and original idea of my native mountains. Yes! I am now convinced that gla-

ciers did descend from the mountains to the plains as they do now in Green land.[11]

Among the younger contemporaries of Agassiz in England, Thomas Henry Huxley appreciated the true value of his work. He once remarked to Marcou:

> Agassiz is a backwoodsman in natural history. He clears up the forest, cutting down all errors, theories, without regard to persons or established reputation. What a pioneer![12]

Yet Huxley found Agassiz's explanation of the origin of species to be unsatisfactory. In a chapter written for Darwin's *Life*, he wrote:

> If *Agassiz* told me that the forms of life which had successively tenanted the globe were the incarnations of successive thoughts of the deity; and that he had wiped out one set of these embodiments by an appalling geological catastrophe as soon as His ideas took a more advanced shape, I found myself not only unable to admit the accuracy of the deductions from the facts of paleontology, upon which this astonishing hypothesis was founded, but I had to confess my want of any means of testing the correctness of his explanation of them. And besides that, I could by no means see that the explanation explained.[13]

Not only in England but in France also Agassiz's friends were numbered among the most prominent scientists. At the *Jardin des Plantes* was Valenciennes, the associate of his early days under the tutelage of Cuvier, who often referred to him as "le cher Agassiz." There also was Professor G. P. Deshayes, who seems to have regretted that Agassiz did not return to the scene of his early triumphs. As late as 1870 he writes to Agassiz:

> What would our old Museum not have gained in having at its head a man like you! We should now not be lying stagnant in a space so insufficient that our buildings, by the mere force of circumstances, are transformed into storehouses, where objects of study are heaped together, and can be of no use to any one. . . . You can fancy how much I envy your organization.[14]

Perhaps the most intimate of Agassiz's Swiss associates were Arnold Guyot and Leo Lesquereux. Guyot and Agassiz had been friends from boyhood. Although separated during their university life, Guyot being at Berlin while Agassiz was at Munich, they became colleagues at Neuchâtel. They studied the Alpine glaciers together, and came to America at about the same time. Both secured positions in American colleges, Agassiz at Harvard and Guyot at Princeton. Throughout their life they shared their scientific interests, being associated in Agassiz's final undertaking at Penikese.

Lesquereux attended the Academy at Neuchâtel with

Agassiz's brother, August. He was a member of the Society of Natural History which Agassiz founded at Neuchâtel. He came to America shortly after Agassiz and became "the foremost North American authority on Paleobotany as well as an able student of Bryology."

Even before coming to America, Agassiz was a correspondent of Benjamin Silliman. As early as 1835 we find Silliman attempting to get subscribers for the *Poissons Fossiles*. After one of Agassiz's early visits to New Haven, Silliman describes him thus:

> He is full of knowledge on all subjects of science, imparts it in the most graceful and modest manner and has, if possible, more of *bonhomie* than of knowledge. He has a more minute knowledge of his subject and at the same time a more wonderful generalizing power and philosophical tone than any man I have ever met.[15]

Silliman could not decide whether the ladies more liked the *man* or the gentlemen more admired the *philosopher*.

Another American scientist with whose work Agassiz was familiar before coming to America was James Hall, the eminent geologist. A sheaf of letters between these two friends cover a period of more than twenty years. In the introduction to his Paleontology Hall acknowledged the help which he received from Agassiz's counsels.

Agassiz early discerned the genius of James Dwight Dana, the son-in-law of Silliman. His friendship with him exerted a powerful influence upon Dana's intellectual growth. The two not only corresponded constantly but also met frequently. Although Dana's letters have not been recovered, "of those received from Agassiz the pile is almost unbroken." Both men were firm theists and ardent glacialists. Both were staunch supporters of the freedom of science from theological restraints. In a letter to Dana in 1856, Agassiz writes:

> I, and we all, are greatly indebted to you for fighting so earnestly the cause of our independence versus clerical arrogance. No one can do it so effectively as you; from me or anyone else who does not profess to be a member of the church it would have no weight with church people at large. I am sorry to find that this clerical spirit is still alive, as bitter, vehement, and overbearing as in the worse times of religious bigotry. It confirms me in my determination to have nothing to do with church matters and church organizations. I do not see but it must come to this, that each and every one must settle religious affairs for himself, without any regard (sic) with others; for, after all, religion is a personal relation to God, and we derive as little comfort from the interference of others with reference to our intercourse with our Maker, as we do in matters of affection.[16]

The coming of Agassiz to America was an inspiration to

another of his younger contemporaries, Spencer Fullerton Baird. One of the first great works projected by Agassiz in America was a memoir upon the fresh-water fishes of North America, in the authorship of which Baird was to have been his associate. However, the work was never completed.

Although Agassiz had few real enemies, he was not equally cordial with all his friends. He had no high opinion of Charles W. Eliot as a scientist, and was largely instrumental in preventing his election to the Rumford professorship of chemistry at Harvard. Another scientist whose ambitions to become a Harvard professor was thwarted by Agassiz was William B. Rogers. Agassiz could not be persuaded to have Rogers beside him in the Lawrence Scientific School. Thus, Rogers turned his energies to creating a rival institution, the Massachusetts Institute of Technology. Asa Gray's efforts to assure Darwin's *Origin* fair play in America strained his friendship with Agassiz. After the appearance of the *Origin* it is said that the students of Agassiz had only a limited contact with Gray or his students. Shaler says that Benjamin Peirce, the mathematician and astronomer, and Agassiz "were enemies, with occasional intermissions of loving friendship." Although Agassiz had incidental frictions with Jeffries Wyman, he always retained friendly relations with him and had a great admiration for him. Agassiz's appreciation of Wyman, who was the very antithesis of Agassiz, led Shaler to believe that Agassiz had a rare capacity for judging men and that his conflicts with others were not due, as some thought, to selfish interests. Nevertheless Agassiz had his share of quarrels with his associates including those with Karl Schimper, Edouard Desor, James D. Forbes, and Henry James Clark.

The foregoing paragraphs are intended to illustrate the various avenues through which Agassiz's influence on his scientific friends was exerted. His personal contacts with Joseph Henry, Samuel George Morton, Leopold von Buch, Edward Forbes, Richard Owen, and many other scientists we have ignored; his correspondence with Sir Philip Egerton, Alexander Braun, Alexander Dallas Bache, Hugh Miller, and Elie de Beaumont and a host of others we have neglected. As it would be impossible to quote tributes from all of Agassiz's associates in science, we shall conclude this section with one in the superlative degree. George B. Emerson, one of the founders of the Boston Society of Natural History says that he found Agassiz to be

"the wisest, the most thoroughly well-informed and communicative, the most warm-hearted and the most modest man of science with whom, personally or by his works," he had ever become acquainted.

3. *Some unknown friends of Agassiz*

Agassiz's influence was exerted through other than Scientific channels. He influenced a great body of unknown students whose tributes were never written. As William James expressed it, Agassiz "left a sort of popular myth—the Agassiz legend, as one might say—behind him in the air about us."

Shaler relates an incident which illustrates Agassiz's personal power over common people. In 1866 Shaler and Agassiz were interested in comparing the skeletons of thorough-bred horses with those of common stock. Shaler had tried without success to get the bones of certain famous stallions from Kentucky. Early one morning the stables at Beacon Park were destroyed by a fire believed to be of incendiary origin. Many horses were badly burned and others were killed. Shaler raced to the track hoping to procure the skeletons of the dead horses. On his arrival, however, he encountered such a mob of irate owners and angry jockeys that he decided to await a more propitious moment to make his request. On his return from the track, he chanced to meet Agassiz who had heard of the fire and was on his way to the Park. Shaler tried to persuade him to turn back assuring him that this was not the time for bargaining for skeletons. Agassiz would not listen and continued to the smouldering ruins. Shaler went along in the hope of protecting him from the consequences of his curiosity. Shaler described the marvel which came about when they reached the spot thus:

> . . . In a moment he had all those raging men at his command. He went at once to work with the horses which had been hurt, but were savable. His intense sympathy with the creatures, his knowledge of the remedies to be applied, his immediate appropriation of the whole situation, of which he was at once the master, made these rude folk at once his friends. Nobody asked who he was, for the good reason that he was heart and soul of them. When the task of helping was done, then Agassiz skilfully came to the point of his business—the skeletons—and this so dextrously and sympathetically, that the men were, it seemed, ready to turn over the living as well as the dead beasts for his service.[17]

Edward Everett Hale was also impressed with Agassiz's influence on ordinary people. He relates one impression thus:

> I have seen him address an audience of five hundred people, not twenty of whom when they entered the hall thought they had anything to do with

the study of nature. And when after his address they left the hall, all of the five hundred were determined to keep their eyes open and to study nature as she is.[18]

Rufus P. Stebbins especially noticed Agassiz's versatility in dealing with all sorts of persons. He describes his observations thus:

> I have heard him pour out the stores of knowledge derived from new investigations before the savants of this continent with the simplicity and self-forgetfulness with which a child would pour its toys into your lap; and I have heard him talk with apparently equal interest before a company of farmers and mechanics, whose knowledge of nature was almost infinitesimal.[19]

The work of Agassiz in encouraging common people to observe nature was continued by The Agassiz Association, founded in 1875 by Harlan H. Ballard, a teacher of Lenox, Massachusetts. In 1888 Ballard reported the total membership to be above ten thousand distributed in all the states and territories with very few exceptions.

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A GEOMETRICAL DISCUSSION OF THE GRAVITATIONAL LAW OF INVERSE SQUARES

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I

Kepler's first law of planetary motion is that the orbit of a planet is an ellipse, at one focus of which is the sun. His second law is that the radius vector between the centers of mass of sun and planet, traverses equal areas in equal times. From these may be deduced and geometrically demonstrated, the law that the force of gravitation between sun and planet, varies inversely as the square of the distance between them. (The second law is more generally discussed in the following section II.)

Throughout this discussion, the centers of mass will be treated as mathematical points wherein the mass is concentrated; and the mass of the sun will be considered as a fixed point. This is justified by the great disparity between the masses.

II

If motion, *momentarily* uniform in a straight line, be accelerated at the ends of equal time-intervals, by impulses equal or unequal, exactly in the direction of an external fixed point, the radius vector from the fixed point to the point defining the motion, will traverse equal areas in equal times. Let O be a fixed point joined by a radius vector to a point B which is momentarily moving with uniform velocity in direction of C. If not deflected, it will travel equal distances in equal times T and will be at equally spaced points B, B_1, B_2 etc. successively. At po-

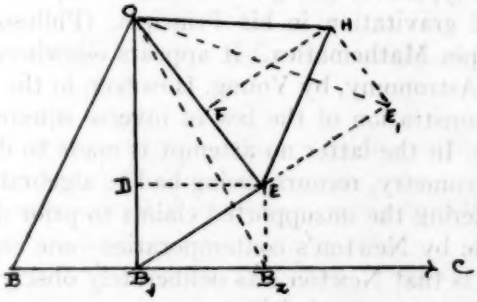

FIG. 1

sition B_1 it receives an impulse exactly directed toward O such that, as the result of that impulse alone, it would move to point D in unit time T. The motion of B will be altered, i.e., accelerated in both speed and direction and its path will be the resultant of its original motion and the superimposed acceleration. Therefore, in unit time T, it will traverse $B_1 E$ (which is the diagonal of the parallelogram of forces) with speed which would carry it to E_1 in the succeeding time T. At point E it receives another impulse directed toward O such that, as the result of that impulse alone, it would move to point F in time T. Again speed and direction are altered and the point will traverse the diagonal $E H$ in time T. (And so for any number of impulses.)

Triangle B_1OB_2 is equal to triangle BOB_1 (having equal bases and the same altitude).

Triangle B_1OB_2 is equal to triangle B_1OE (having same base (B_1O) and equal altitudes).

Therefore triangle BOB_1 is equal to triangle B_1OE

Triangle EOE_1 is equal to triangle B_1OE (having equal bases and same altitude).

Triangle EOE_1 is equal to triangle EOH (having same base OE and equal altitudes).

Therefore triangle B_1OE is equal to triangle EOH

And so for all areas similarly defined. Notice that the impulses may be equal or unequal. The important thing to notice is the equality of areas.

Substantially, the foregoing demonstration is part of Newton's discussion of gravitation in his Principia. (Philosophiae Naturalis Principia Mathematica.) It appears elsewhere, as e.g., in the General Astronomy, by Young. However, in the former, the ensuing demonstration of the law of inverse squares is regretably obscure. In the latter no attempt is made to demonstrate the law by geometry, recourse being had to algebraic equations only. Considering the unsupported claims to prior discovery of the law made by Newton's contemporaries—one especially—it is fair to guess that Newton was deliberately obscure. So far as other works are concerned, I have never come across a demonstration. Authors seem to have been content to state the law and ascribe its discovery to Newton.

III

Let F_1 be a focus of the elliptical orbit of point (center of mass) P, with the center of mass of the sun at F_1. Let F_2 be the empty

focus. P and p are two positions of the planet moving with instantaneous velocities tangent to the orbit. Consider that in unit time T, gravitation deflects P from the tangent through the distance D and deflects p through distance d. R and r are corresponding positions of the radius vector F_1P and F_1p. The areas traversed by the vector are triangles when T is very short. Let the bases of the triangles be B and b. The areas of the triangles

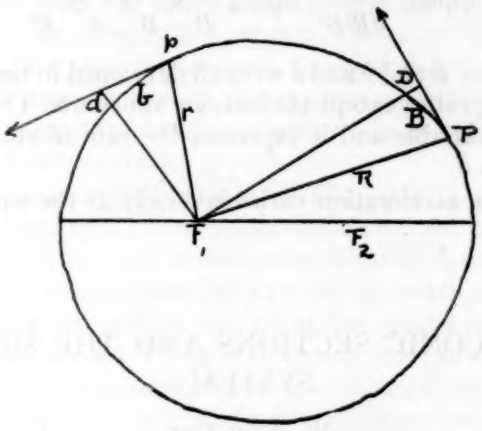

FIG. 2

are equal. (See II.) Call the area K. Let A and a designate radial acceleration.

$$D \text{ equals } \frac{AT^2}{2} \quad \text{and} \quad d \text{ equals } \frac{aT^2}{2}.$$

$$\text{Whence} \quad T^2 \text{ equals } \frac{2D}{A} \quad \text{and} \quad T^2 \text{ equals } \frac{2d}{a} \tag{1}$$

$$K \text{ equals } \frac{RB}{2} \quad \text{and} \quad K \text{ equals } \frac{rb}{2}.$$

(Where T is very short & R & r may be considered to be altitudes of ▲),

$$KT \text{ equals } \frac{RBT}{2} \quad \text{and} \quad KT \text{ equals } \frac{rbT}{2}$$

$$K^2T^2 \text{ equals } \frac{R^2B^2T^2}{4} \quad \text{and} \quad K^2T^2 \text{ equals } \frac{r^2b^2T^2}{4}.$$

$$\text{Whence} \quad T^2 \text{ equals } \frac{R^2B^2T^2}{4K^2} \quad \text{and} \quad T^2 \text{ equals } \frac{r^2b^2T^2}{4K^2} \tag{2}$$

Equating (1) and (2),

$$\frac{R^2B^2T^2}{4K^2} \text{ equals } \frac{2D}{A} \quad \text{and} \quad \frac{r^2b^2T^2}{4K^2} \text{ equals } \frac{2d}{a}.$$

Whence A equals $\frac{8K^2D}{R^2B^2T^2}$ and a equals $\frac{8K^2d}{r^2b^2T^2}$.

Therefore $\frac{A}{a}$ equals $\frac{Dr^2b^2}{dR^2B^2}$ equals $\frac{D}{B}\times\frac{b}{B}\times\frac{b}{d}\times\frac{r^2}{R^2}$.

Because D and B and b and d were all developed in time T, all of the foregoing ratios except the last, are constants. The last ratio is the only variable and it expresses the rate of change in the ratio of A to a.

Therefore the acceleration varies inversely as the square of the distance.

THE CONIC SECTIONS AND THE SOLAR SYSTEM

W. Roy Utz

University of Virginia, Charlottesville, Va.

The geometry of the solar system is a happy discovery for the teacher of geometry searching for illustrations to enliven the conventional material in either synthetic or analytic geometry. Indeed, astronomy provides a host of illustrations and applications of various branches of mathematics that are both interesting and neglected. The neglect is due largely to the widespread lack of information, among otherwise well-educated people, of even the bare fundamentals of astronomy. Partially this is because many astronomers have an unfortunate lack of interest in the teaching of astronomy. There seem to be some understandable reasons for this situation, not the least of which is the fascination that they themselves find in the non-teaching aspect of their work. Furthermore the equipment necessary for the teaching of observational astronomy is expensive, necessarily difficult to reach and often needed for pressing research.

It has been the author's practice during the past several years to include in courses in analytic geometry a lecture on the occurrences of the conic sections in the motion of the members of the solar system. In even the dullest classes this has caused an extra-

ordinary interest toward both the geometry and astronomy presented.

Of primary importance in such a presentation are the nine known planets. They, together with the minor planets or asteroids (of which over 1,500 are known), have elliptic orbits with the sun at one focus. The closest and farthest points from the sun in each orbit are called the perihelion and aphelion points, respectively. They, of course, represent the endpoints of the major axis of the ellipse. The motion of a planet in its orbit is periodic. In the case of the earth the period is one (sidereal) year. At the same time each planet rotates on its axis. In the case of the earth this rotation is effected in twenty-four hours (less about four minutes).

In the accompanying TABLE OF PLANETS the planets are arranged according to their mean distance from the sun. The semi-major axis is given in astronomical units (one unit being, by definition, the semi-major axis of the earth's orbit), the period is given in sidereal years and the orbital velocity is in kilometers per second. Using information from this table a student can compute the perihelion and aphelion distance from the sun, the length of the minor axis of the ellipse, etc.

TABLE OF PLANETS

Name	Semi-Major Axis of Orbit	Period	Mean Orbital Velocity	Eccentricity
Mercury	0.387	0.24	47.9	0.206
Venus	0.723	0.62	35.1	0.007
Earth	1.000	1.00	29.8	0.017
Mars	1.524	1.88	24.1	0.093
Jupiter	5.203	11.86	13.1	0.048
Saturn	9.539	29.46	9.7	0.056
Uranus	19.191	84.01	6.8	0.047
Neptune	30.071	164.78	5.4	0.009
Pluto	39.518	248.42	4.7	0.247

Most of the planets have satellites. The motion of the earth's satellite, the moon, is still the subject of considerable research. However, for our purpose its orbit can be taken as an ellipse with the earth at one focus and having eccentricity about 0.056.

In addition to the sun, the planets with their satellites, and the minor planets (asteroids), the solar system consists of numerous comets and meteors. The orbits of about 400 comets

have been studied; of these the majority seem to be parabolic, many are elliptic and a few are hyperbolic. In each case the sun is at a focus of the orbit and the terms perihelion and aphelion apply as before. Of course only the comets with elliptic orbits remain in the solar system. The information available concerning the orbits of comets is relatively scarce since, with a few exceptions, the object is observable for only a comparatively short period of time. However in many cases of periodic (i.e. elliptic) orbits there have been sufficient observations to establish more than the nature of the orbit. Because of their tendency to visit the vicinity of bodies of large mass (notably the sun and Jupiter) their orbits are frequently altered in a radical manner.

Very little is known about the orbits of meteors since, for the most part, they make themselves known by entering the earth's atmosphere where they burn themselves out or become embedded in the earth as meteorites. A few, however, are large enough to be observed in their orbits and others seem to follow the orbits of "dead" comets of which they were probably a part. In these cases the orbits are elliptic with the sun at one focus.

A discussion of this nature should not omit the use made of a property of the parabola in the construction of reflecting telescopes. The essential part of a reflecting telescope is a mirror, the surface of which is a parabola of revolution with vertex at the center of the mirror. The light from an object to be observed comes to the surface of this mirror in rays essentially parallel to the axis of the parabola. Due to the parabola's property of reflecting such a ray through its focus, a concentration of light from the object is found at the focus of the parabola. A small convex mirror, placed at the focus of the parabola, reflects the rays back through a hole in the center of the large mirror where there has been placed an eyepiece or photographic plate. The mirror of the largest reflector in the world (Mt. Wilson) is 100-inches in diameter. The Palomar reflector will have a mirror 200-inches in diameter.

The reader who wants a further reference will find *Astronomy, I, The Solar System*, by Russell, Dugan and Stewart (Ginn and Co., 1945) both readable and authoritative.

A modern nation is not stronger than its progress in metals. In that progress America leads the world.

Thomas V. Carlisle *Your Career in Engineering*

WORLD CENTER ON MINOR PLANETS ESTABLISHED AT CINCINNATI OBSERVATORY

So far as minor planets are concerned, Cincinnati Observatory has become the center of the solar system. The International Astronomical Union has designated it as the world's collecting and distributing center for observations on asteroids or minor planets.

Astronomers from all parts of the world will send to Cincinnati observations, ephemerides, and new discoveries, which will then be bulletined by wire, cable and mail to all observatories.

Cincinnati Observatory, headed by Dr. Paul Herget, will thus serve the same function with regard to minor planets as Harvard College Observatory, Cambridge, Mass., does in the distribution of general astronomical data by means of its famous series of announcement cards and telegrams.

A NEW METHOD FOR EXTRACTION OF ALGEBRAICAL SQUARE ROOT

ST. KAIDY TAN

Chip-Bee Senior Middle School, Amoy, China

This article on a method for finding the square root of any algebraic integral expression will be found useful and interesting to many readers.

First, let us adopt the example and method as shown in Chrystal's *Algebra*, I, p. 217:

Find the square root of $x^{10}+6x^9+13x^8+4x^7-18x^6-12x^5+14x^4-12x^3+9x^2-2x+1$.

The process is thus:

$$
\begin{array}{r|l}
 & 1+3+2-4+1-1 \\
\hline
 & 1+6+13+4-18-12+14-12+9-2+1 \\
 & 1^* \\
\hline
(2)+3 & \quad 6^*+13^* \\
 & \quad 6^*+9 \\
\hline
2+(6)+2 & \qquad 4+4^*-18^* \\
 & \qquad 4^*+12+4 \\
\hline
2+6+(4)-4 & \qquad\quad -8-22-12^*+14^* \\
 & \qquad\quad -8^*-24-16+16 \\
\hline
2+6+4-(8)+1 & \qquad\qquad 2+4-2-12^*+9^* \\
 & \qquad\qquad 2^*+6+4-8+1 \\
\hline
2+6+4-8+(2)-1 & \qquad\qquad -2-6-4+8-2^*+1^* \\
 & \qquad\qquad -2^*-6^*-4^*+8^*-2^*+1^* \\
\hline
\end{array}
$$

We observe that the numbers marked with an asterisk are the same as the numbers in the same column upright opposite to

them, it is unnecessary to write out repeatedly; and the numbers on the left side of the vertical line it is required only to write out those in the parentheses. Hence the above operation may be shortened as follows:

$$
\begin{array}{r|l}
 & 1+3+\ 2-\ 4+\ 1-\ 1 \\
\hline
2 & 1+6+13+\ 4-18-12+14-12+9-2+1 \\
 & \quad 9 \\
 & \quad - \\
6 & \quad\ 4+12+\ 4 \\
 & \quad\ - \\
 & \quad -8-22 \\
4 & \quad\ -24-16+16 \\
 & \quad\ - \\
 & \quad\ \ 2+\ 4-\ 2 \\
-8 & \quad\ \ \ 6+\ 4-\ 8+\ 1 \\
 & \quad\ \ - \\
2 & \quad\ \ -2-6-4+8
\end{array}
$$

Ans. $\pm(x^5+3x^4+2x^3-4x^2+x-1)$.

If the readers are skilful in mental arithmetic, then the process can be simplified more briefly as follows:

$$
\begin{array}{r|l}
 & 1+3+\ 2-4+\ 1-\ 1 \\
\hline
2 & 1+6+13+4-18-12+14-12+9-2+1 \\
6 & \quad 4-8-22 \\
4 & \quad\ \ 2+\ 4-\ 2 \\
-8 & \quad\ \ -2-6-4+8 \\
2 &
\end{array}
$$

Another Example: Find the square root of $4x^4-12x^3+25x^2-16x+9$.

Solution:

$$
\begin{array}{r|l}
 & 2-\ 3+\ 4 \\
\hline
4 & 4-12+25-16+\ 9 \\
 & \quad\ 9 \\
 & \quad - \\
-6 & \quad -16-24+16 \\
 & \quad\ \ - \\
 & \quad\ \ 8-\ 7
\end{array}
\qquad \text{or:} \qquad
\begin{array}{r|l}
 & 2-\ 3+\ 4 \\
\hline
4 & 4-12+25-16+9 \\
-6 & \quad -16+\ 8-7
\end{array}
$$

As we extract the root of the given expression, it remains $8x-7$; consequently, we know the radicand is not a perfect square.

Kitchen dishtowel, 20% asbestos and 80% cotton, wipes dishes faster and drier because asbestos has ability to absorb considerable quantities of water rapidly. Also the asbestos, which is a mineral, imparts a polish to china and glass.

PROBLEM DEPARTMENT

CONDUCTED BY G. H. JAMISON
State Teachers College, Kirkville, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution or proposed problem sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to G. H. Jamison, State Teachers College, Kirksville, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solution.

2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

3. In general when several solutions are correct, the one submitted in the best form will be used.

LATE SOLUTIONS

2023, 4, 6, 7, 8. *St. Kaidy Tan, Chip-Bee Institute, Amoy China.*

2025. *Philip Rosenblatt, Brooklyn.*

2040. *M. Kirk, Philadelphia.*

2029. *Proposed by Lillian F. Hayden, Troy, N. Y.*

If in triangle ABC, $a^4+b^4+c^4=2c^2(a^2+b^2)$, find angle C.

Solution by Max Beberman, Nome, Alaska

By the Law of Cosines:

$$2ab\cos C=a^2+b^2-c^2.$$

Squaring both sides:

$$4a^2b^2\cos^2 C=a^4+b^4+c^4-2c^2(a^2+b^2)+2a^2b^2.$$

Substituting the given relation in the right hand member:

$$4a^2b^2\cos^2 C=2a^2b^2.$$

Therefore

$$\cos^2 C=\tfrac{1}{2}.$$

Hence

$$C=45°, 135°.$$

Solutions were also offered by Lillian F. Hayden, Troy, N. Y.; Mildred Hopkins, Kankakee, Ill.; M. Kirk, Philadelphia; Raymond Kassler, Forest Hills, N. Y.; V. C. Bailey, Evansville, Md.; Hugo Brand, Chicago; Mrs. L. E. Gurley, Troy, N. Y.; Louise Bolander, Birmingham, Ala.;

Walter R. Warne, Dayton, Ohio; Hazel S. Wilson, Annapolis; Efrain Pinell C.; L. R. Galesburg, Lebanon, Pa.; Clinton A. Bergstresser, Brooklyn, N. Y.; Herbert J. Stepaniak; B. Felix John, Philadelphia.

2030. *Proposed by James S. Frost, Willard, N. Y.*

Solve

$$x(y+z)=a$$
$$y(x+z)=b$$
$$z(x+y)=c.$$

Solution by Clinton A. Bergstresser, Brooklyn, N. Y.

$$xy+xz=a \tag{1}$$
$$xy+yz=b \tag{2}$$
$$xz+yz=c. \tag{3}$$

By addition and division by 2

$$xy+xz+yz=\frac{a+b+c}{2} \tag{3}$$
$$yz=\frac{-a+b+c}{2} \tag{4}$$
$$xz=\frac{a-b+c}{2} \tag{5}$$
$$xy=\frac{a+b-c}{2} \tag{6}$$
$$x^2y^2z^2=\frac{(-a+b+c)(a-b+c)(a+b-c)}{8} \tag{7}$$
$$xyz=\pm\sqrt{\frac{(-a+b+c(a-b+c)(a+b-c)}{8}}. \tag{8}$$

By easy substitution

$$x=\pm\sqrt{\frac{(a-b+c)(a+b-c)}{2(-a+b+c)}}$$
$$y=\pm\sqrt{\frac{(-a+b+c)(a+b-c)}{2(a-b+c)}}$$
$$z=\pm\sqrt{\frac{(-a+b+c)(a-b+c)}{2(a+b-c)}}.$$

Solutions were also offered by B. Felix John, Philadelphia; M. Kirk, Philadelphia; Joseph Lerner, Roxbury, Mass.; Daniel Block, N.Y.; Raymond Kassler, Forest Hills, N. Y.; Clara Lane, Lexington, Ky.; Betty L. Preister, Baton Rouge, La.; Namoi Meegraw, Covington, Ky.; Sidney Bell, Portland, Me.; Walter R. Warne, Dayton, Ohio; Hugo Brandt, Chicago; Max Beberman, Nome, Alaska.

2031. *Proposed by Hugo Brandt, Chicago.*

Derive a formula to show that there are 33 pairs of consecutive primes between the numbers 11 and 1000 inclusive. Also find them.

Hugo Brandt with much head and very much hand labor found the 33 pairs but no formula. Only an outline will be given to show his method.

Within the limits the primes are all odd and of the form $2n+1$. To "weed out" the composite numbers set $n=3a\pm(0,1)$, n positive. This leads to the possibilities, $6a+(\pm1,3)$ or, since $6a+3$ is not prime, to the form $6a\pm1$. This involves multiples of 5. To eliminate them set $a=5b\pm(0,1,2)$ and the required primes are found in $30b\pm(1,5,7,11,13)$. The weeding out process of multiples of 5, 7, 11, 13, 17, 19, 23, 29 is continued and finally with great labor the 33 pairs are found.

2032. *Proposed by Susan Warne, St. Johns, Newfoundland.*

Show that the series

$$\frac{1}{2}+\frac{1}{1+\sqrt{2}}+\frac{1}{1+\sqrt{3}}+\cdots$$

is divergent.

Solution by V. C. Bailey, Evansville, Indiana

The following harmonic series is divergent.

$$\frac{1}{1+1}+\frac{1}{1+2}+\frac{1}{1+3}+\cdots.$$

The given series, term by term, is greater than the known divergent series. Also

$$\frac{1}{1+\sqrt{n}}>\frac{1}{1+n},$$

therefore, the given series is divergent.

Other solutions were given by: Walter R. Warne, Dayton, Ohio; M. Kirk, Philadelphia; Wayne R. Cowell, Manhattan, Kan.; Joseph Lerner, Roxbury, Mass.; Daniel Block, New York; Hugo Brandt, Chicago; Norman Anning, University of Michigan.

2033. *Proposed by Norman Anning, University of Michigan.*

If a and n are positive integers, prove that $(\sqrt{a}+\sqrt{a}+\sqrt{a+1})^n$ is the sum of the square roots of two consecutive positive integers.

Solution by the proposer

I We have

$$(\sqrt{a+1}+\sqrt{a})^2=2a+1+2\sqrt{a^2+a}=\sqrt{4a^2+4a+1}+\sqrt{4a^2+4a}.$$

Similarly

$$\begin{aligned}(\sqrt{a+1}+\sqrt{2})^3&=(4a+1)\sqrt{a+1}+(4a+3)\sqrt{a}\\&=\sqrt{16a^3+24a^2+9a+1}+\sqrt{16a^3+24a^2+9a}.\end{aligned}$$

Thus the theorem is true for $n=2$ and for $n=3$.

II. Suppose the theorem is true for some exponent k, an *odd* positive integer. Then there exist integers r and s such that

$$(\sqrt{a+1}+\sqrt{a})^k=r\sqrt{a+1}+s\sqrt{a}$$

and such that

$$r^2(a+1)=1+s^2a. \tag{1}$$

Multiply both sides by $(\sqrt{a+1}+\sqrt{a})$.

$$(\sqrt{a+1}+\sqrt{a})^{k+1}=(ra+r+sa)+(s+r)\sqrt{a+a^2}.$$

We now use (1) in showing that

$$(ra+r+sa)^2=1+(s+r)^2(a+a^2).$$

$$\begin{aligned}1+(s+r)^2(a+a^2)&=1+s^2a+2rsa+r^2a+s^2a^2+2rsa^2+r^2a^2\\&=r^2a+r^2+2rsa+r^2a+s^2a^2+2rsa^2+r^2a^2\\&=(ra+r+sa)^2.\end{aligned}$$

It follows that the truth of the theorem for k, odd, implies the truth for $(k+1)$.

III. Suppose the theorem is true for some exponent k, an *even* positive integer. Then there exist integers p and q such that

$$(\sqrt{a+1}+\sqrt{a})^k=p+q\sqrt{a+a^2}$$

and such that

$$p^2=1+q^2a+q^2a^2. \tag{2}$$

We would have

$$(\sqrt{a+1}+\sqrt{a})^{k+1}=(p+qa)\sqrt{a+1}+(p+qa+q)\sqrt{a}.$$

And, by the method used in II, it is easy to show that

$$(p+qa)^2(a+1)=1+(p+qa+q)^2a.$$

It follows that the truth of the theorem for k, even, implies the truth for $(k+1)$.

IV. Since k must be odd or even, the proof is complete.

A solution by Mildred Hopkins, Kankakee, Ill. was offered.

2034. *No satisfactory solution has been offered.*

HIGH SCHOOL HONOR ROLL

The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each high school contributor will receive a copy of the magazine in which the student's name appears.

For this issue the Honor Roll appears below.

2029, 2032. *John J. Kaplan, Medwood H. S.; Brooklyn.*

PROBLEMS FOR SOLUTION

2053. *Proposed by W. R. Warne, Dayton, Ohio.*

Solve the system:

$$\begin{aligned}y^2&=xz\\x+y+z&=28\\yxz&=512.\end{aligned}$$

2054. *Proposed by Hugo Brandt, Chicago.*

Find the area of the loop of $y^2(1+x)=x^2(1-x)$.

2055. *Proposed by William W. Johnson, Cleveland, Ohio.*

Solve the equation, and give a solution adapted to logarithmetic computation

$$84\sin(14^\circ-x)-86\sin x=3.$$

2056. *Proposed by William W. Johnson, Cleveland, Ohio.*

A metal cube, edge 4, has holes with 2 as diameter bored through the center of all its faces. What is the volume of the metal remaining in the cube?

2057. *Proposed by Walter R. Warne, Dayton, Ohio.*

Solve without use of the factor theorem and synthetic division:

$$x^4-6x^3+12x^2-20x-12=0.$$

2058. *Proposed by Norman Anning, University of Michigan.*

Show by skill rather than strength that

$$\begin{vmatrix} a & a+p & a+2p \\ b+2q & b+q & b \\ c & c+r & c+2r \end{vmatrix} = 0.$$

BOOKS AND PAMPHLETS RECEIVED

THE MATHEMATICAL THEORY OF FINANCE, by Kenneth P. Williams, Professor of Mathematics, Indiana University. Revised Edition. Cloth. Pages x+274. 14×21 cm. 1947. The Macmillan Company, 60 Fifth Avenue, New York. Price $4.50.

ARITHMETIC, by Clifford B. Upton and Kenneth G. Fuller. Grade Eight. Cloth. Pages vii+360. 13×20 cm. 1947. American Book Company, 88 Lexington Avenue, New York 16, N. Y.

LECTURE EXPERIMENTS IN CHEMISTRY, by G. Fowles, M.Sc., A.R.I.C., F.C.S. (England). Third Edition with Supplement. Cloth. Pages xvi+612. 13.5×21 cm. 1947. The Blakiston Company, 1012 Walnut Street, Philadelphia 5, Pa. Price $7.00.

BASIC BIOLOGY FOR HIGH SCHOOLS, by Carroll Lane Fenton, Ph.D., and Paul E. Kambly, Ph.D., Professor of Education and Director of Supervised Teaching, University of Oregon. Cloth. Pages ix+726. 15×20.5 cm. 1947. The Macmillan Company, 60 Fifth Avenue, New York, N. Y. Price $3.24.

CALCULUS, by G. E. F. Sherwood, Professor of Mathematics, and Angus E. Taylor, Associate Professor of Mathematics, University of California at Los Angeles. Revised Edition. Cloth. Pages xiv+568. 15×23 cm. 1946. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. Price $3.75.

COLLEGE ALGEBRA, by M. Richardson, Ph.D., Assistant Professor of Mathematics, Brooklyn College. Cloth. Pages xvi+472. 14.5×22.5 cm. 1947. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. Price $2.85.

THE STATUS OF THE BEGINNING CALCULUS STUDENTS IN PRECALCULUS COLLEGE MATHEMATICS, by Mary Draper Boeker, Ph.D., Instructor in Mathematics, Brooklyn College. Cloth. Pages viii+83. 15×22.5 cm. 1947. Bureau of Publications, Teachers College, Columbia University, New York, N. Y. Price $2.15.

CHILDREN'S CONTRIBUTIONS IN SCIENCE DISCUSSIONS, by Katherine Elizabeth Hill, Ph.D. Contribution to Education, No. 931. Cloth. Pages

viii+96. 15×22.5 cm. 1947. Bureau of Publications Teachers College, Columbia University, New York, N. Y. Price $2.10.

ROMPING THROUGH MATHEMATICS, by Raymond W. Anderson. Cloth. 152 pages. 15×20.5 cm. 1947. Alfred A. Knopf, Publisher, 501 Madison Avenue, New York 22, N. Y. Price $2.50.

SECONDARY-SCHOOL SCIENCE TEACHING, by Arthur G. Hoff, Ph.D., Associate Professor of Education, University of Redlands, Redlands, Calif. Cloth. Pages xi+303. 14.5×21.5 cm. 1947. The Blakiston Company, 1012 Walnut Street, Philadelphia, 5, Pa. Price $3.75.

ADVENTURES WITH PLASTICS, by Louis V. Newkirk, Ph.D., Director, Bureau of Industrial Arts; Coleman Hewitt, M.A., Chicago Teachers College; and LaVada Zutter, M.A., Designer and Illustrator, and Former Teacher Arts and Crafts, Chicago Public Schools. Cloth. Pages x+275. 18×25.5 cm. 1947. D.C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $3.50.

NOW TRY THIS, by Herman and Nina Schneider. Paper. 40 pages. 20.5×25 cm. 1947. William R. Scott, Inc., 513 Avenue of the Americas, New York 11, N. Y. Price $1.50.

EDUCATION IN ECUADOR, by Cameron D. Ebaugh, Senior Specialist on Education in Latin American Countries, American Republics Section, Division of International Educational Relations. Bulletin 1947. No. 2. Pages vi+92. 14.5×23.5 cm. Superintendent of Documents, U. S. Printing Office, Washington 25, D. C. Price 25 cents.

EDUCATION IN EL SALVADOR, by Cameron D. Ebaugh, Senior Specialist on Education in Latin American Countries, American Republics Section, Division of International Education Relations. Bulletin 1947, No. 3. Pages vi+81. 14.5 ×23.5 cm. Superintendent of Documents, U. S. Printing Office, Washington 25, D. C. Price 25 cents.

1947 REPORT OF THE PROFESSIONAL ETHICS COMMITTEE OF THE NEA. Paper. 64 pages. 15×23 cm. National Education Association Headquarters Office, 1201 Sixteenth Street, N. W., Washington 6, D. C.

ANNUAL REPORT OF THE NATIONAL FOUNDATION FOR INFANTILE PARALYSIS FOR THE FISCAL PERIOD JUNE 1, 1946 TO MAY 31, 1947. Paper. Pages vi+86. 14×21.5 cm. The National Foundation for Infantile Paralysis, 120 Broadway, New York 5, N. Y.

PRACTICAL EXERCISES IN BUSINESS ARITHMETIC, by James L. Twohig, *Teacher of Bookkeeping, William Penn High School, Philadelphia, Pa.* Paper. Pages iii+124. 20.5×27 cm. 1947. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price 72 cents.

PORTRAITS OF EMINENT MATHEMATICIANS WITH BRIEF BIOGRAPHICAL SKETCHES, by David Eugene Smith, *Professor Emeritus of Mathematics, Columbia University.* Portfolio, Number One. New Deluxe Edition. 25×24 cm. 1946. Scripta Mathematica, Yeshiva University, 186th Street and Amsterdam Avenue, New York, N. Y. Price $5.00.

WORKBOOK FOR SCIENCE ON THE MARCH, Book One, by Curtis Love, *Head of the Science Department, Pine Bluff High School, Pine Bluff, Arkansas;* and John A. Clark, *Head of Department of Physical Sciences, Alexander Hamilton High School, Brooklyn, New York.* Paper. 170 pages. 19.5×28

cm. 1947. Houghton Mifflin Company, 2 Park Street, Boston, Mass. Price 96 cents.

INTER-AMERICAN UNDERSTANDING AND THE PREPARATION OF TEACHERS, by Effie G. Bathurst, *Supervisor, Inter-American Teacher-Education Programs*. Bulletin 1946, No. 15. Pages iv+100. 14+23 cm. Superintendent of Documents, United States Government Printing Office, Washington 25, D. C. Price 30 cents.

SEVEN RULES OF CLEAR THINKING, by Roy C. Bryan, Assisted by Leonard Gernant, A. L. Sebaly, George E. Mills, and Robert A. Bahlman. Paper 78 pages. 21.5×28 cm. 1947. Roy C. Bryan, Western Michigan College, Kalamazoo, Mich.

HEALTH INTERESTS OF CHILDREN. Report of a Research Study of Health Interests and Needs of Children as a Basis for Health Instruction—Kindergarten Through Grade Twelve. Paper. Pages x+121. 15×23 cm. 1947. Department of Instruction, Denver Public Schools, Denver 2, Colo. Price $1.25.

THE MANUAL OF STANDARDS AND DIRECTORY OF PRIVATE HOME STUDY SCHOOLS AND COURSES. J. S. Noffsinger, Editor. Paper. 48 pages. 13.5×20 cm. 1947. The National Home Study Council, 839 17th Street, N.W., Washington 6, D. C.

CAREER OPPORTUNITIES IN AVIATION. A HANDBOOK OF VOCATIONAL INFORMATION. Paper. 31 pages. 14×21.5 cm. National Council of Technical Schools, 839 17th Street, N.W., Washington 6, D. C. Price 15 cents.

EMPLOYMENT OPPORTUNITIES IN THE NAVY DEPARTMENT FOR SCIENTISTS AND TECHNICIANS. Paper. 82 pages. 20×27 cm. March, April, May, 1947. Navy Department, Office of Naval Research, Scientific Personnel Branch, Washington 25, D. C.

LIFE THROUGH THE AGES. A VISUAL INTRODUCTION TO THE STORY OF CHANGE IN LIVING THINGS, by R. Will Burnett, *Chairman of the Department of Natural Sciences, San Francisco State College*. Paper. 48 pages. 18.5×26 cm. Stanford University Press, Stanford University, Calif. Price $1.00.

THE SUGAR MOLECULE. Quarterly Review of Sugar Research. July, 1947. Vol. 1, No. 1. Sugar Research Foundation, Inc., 52 Wall Street, New York, N. Y.

CHEMISTRY, PHYSICS, MATHEMATICS ENGINEERING, MEDICINE. A Catalog Listing all Scientific Publications of Interscience Publishers, Inc., 48 pages. 15×22.5 cm. 1947. Interscience Publishers, Inc., 215 Fourth Avenue, New York 3, N. Y.

DOVER PUBLICATIONS. Catalog of Scientific Books, 1947. 36 pages. 14×21.5 cm. Dover Publications, 1780 Broadway, New York 19, N. Y.

PROCEEDINGS OF THE THIRTY-EIGHTH, THIRTY-NINTH AND FOURTIETH ANNUAL MEETINGS (INCLUDING 1946 SPRING MEETING) OF LIFE INSURANCE ASSOCIATION OF AMERICA. Paper. 182 pages. 14.5×22 cm. New York N. Y.

EQUIPMENT AND SUPPLIES—BIOLOGICAL AND ENTOMOLOGICAL. This Catalog Lists Over 400 Items. 43 pages. 21.5×28 cm. 1947. Ward's Natural Science Establishment, Inc., P. O. Box 24, Beechwood Station, Rochester 9, N. Y.

Publications of the National Roster of Scientific and Specialized Personnel. March 1947.

Pamphlet No. 1, Agricultural and Biological Sciences. Pages iii+39. 15×23 cm. Price 15 cents.

Pamphlet No. 2, Engineering Sciences. Pages iii+51. 15×23 cm. Price 15 cents.

Pamphlet No. 3, Medical Service Occupations. Pages v+9. 15×23 cm. Price 5 cents.

Pamphlet No. 4, Medical Professions. Pages iii+20. 15×23 cm. Price 10 cents.

Pamphlet No. 5. Miscellaneous Professional Fields. Pages v+29. 15×23 cm. Price 10 cents.

Pamphlet No. 6, Physical Sciences. Pages iv+20. 15×23 cm. Price 10 cents.

Pamphlet No. 7, Social Sciences. Pages iii+31. 15×23 cm. Price 10 cents.

Superintendent of Documents, Government Printing Office, Washington 25, D. C.

STREAMLINED METHODS OF COMPUTING WITH SLIDE RULE AND MATHEMATICAL TABLES, by Donald V. Mitchell. Paper. 80 pages. 13.5×21.5 cm. 1947. Donald V. Mitchell, 5501 30th Avenue N.E., Seattle 5, Wash. Price $1.00.

SIX-PLACE TABLES FOR PRECISION COMPUTING TO ACCOMPANY "STREAMLINED METHODS OF COMPUTING WITH SLIDE RULE AND MATHEMATICAL TABLES," by Donald V. Mitchell. Paper. 47 pages. 13.5×21.5 cm. 1945. Donald V. Mitchell, 5501 30th Avenue N.E., Seattle 5, Wash. Price $1.00.

EDUCATORS GUIDE TO FREE FILMS, Compiled and Edited by Mary Foley Horkheimer, and John W. Diffor, M.A., *Visual Education Director, Randolph High School, Randolph, Wisconsin. Educational Consultant;* John Guy Fowlkes, Ph.D., *Professor of Education and Dean of Summer Session, University of Wisconsin.* Seventh Edition. Paper. Pages xi+341. 20×27.5 cm. Revised, 1947. Educators Progress Service, Randolph, Wis. Price $5.00.

DISQUISITIONES MATHEMATICAE ET PHYSICAE. Tomus IV. Fasc. 1–4. Pages 240, and Tomus V. Fasc. 1–4. Pages 172. Paper. 17×25 cm. Monitorul Oficial Şi Imprimerile Statuli Imprimeria Nationala, Bucuresti, România.

BOOK REVIEWS

SCIENCE—ITS EFFECT ON INDUSTRY, POLITICS, WAR, EDUCATION, RELIGION AND LEADERSHIP, by D. W. Hill, D. Sc. Cloth. Pages v+114, 14×22 cm. 1946. Chemical Publishing Co., Inc. Brooklyn, N. Y. Price $2.75.

This book is a reassuring one in light of the recent attacks on science and the Scientific method. The author points out that even though Science has exploited and given us control over our environment, the real function of science is to understand. Some may call this an age of gadgets produced by science, but one of the chief contributions of science has been a method of thought superior to none—the scientific method. The book points out that science and Religion, having fought a battle in the past, are not nearly as

far apart as one thinks. Science and religion must work more closely together if mankind is to benefit. Leadership in the affairs of men has been given mostly to those trained in the humanities—now it is time to produce men trained both in the liberal arts and science. Only by this means can education give to the world, men both exact and cultural in their outlook. The book points out the close dependence of industry and politics on science. It is true, as the author points out, that science has contributed many weapons of war; but it also points out that war is not the responsibility of science. That responsibility rests on every citizen. A brick can be used for a door stop or for a weapon. Science is neither good nor bad—it is what man does with science or its products that may be good or bad.

The book points out that the future of the country depends on an educated citizenry. Whatever the question or problem that arises, it must be solved. Science can contribute to the education of children by helping them to think through the problems that confront them in school and in life. Throughout the book, the author shows that if the method of science were applied to the problems confronting man in religion, politics, education and leadership, man would use the products of science to improve social and economic progress. We need only quote Benjamin Franklin to substantiate Dr. Hill: "The rapid progress true science now makes, occasions my regretting sometimes that I was born so soon. It is impossible to imagine the height to which may be carried, in a thousand years, the power of man over matter. O that moral science were in a fair way of improvement, that men would cease to be wolves to one another, and that human beings would at length learn what they now improperly call humanity." Dr. Hill's book is an outstanding contribution to the most significant problem now facing the world.

KENNETH E. ANDERSON

BASIC CHEMISTRY, by Ernest E. Bayles, Professor of Education, University of Kansas; and Arthur L. Mills, Chairman, Department of Science, Lake Geneva High School, Lake Geneva, Wisconsin. Cloth. Pages xii+720, 15×22 cm. 1947. The Macmillan Company, New York, N. Y. Price $3.00.

Here is a good chemistry text that attempts to secure a sequential arrangement of each unit so that one unit prepares for an understanding of the next. Kindred topics are more closely grouped together than in the conventional text. The "tool" topics of chemistry are arranged in a logical fashion that should lead to a better understanding on the part of the pupil. Atomic structure and subatomic structure are given a thorough treatment—more than in any text I have seen for some time. "The Questions for Review and Further Study" are comprehensive and good. Each chapter has a section called "Books for Further Reading." The teacher will find these two aids very valuable. Although the book lists some experiments for laboratory work, the teacher will need to supplement this phase to some extent. This book, as do many, attempts to treat the scientific method of thinking, but fails to do it in a manner that will carry over into the thinking of pupils. Perhaps this still has to be taught by inspired teachers.

KENNETH E. ANDERSON

DIRECTED EXPERIMENTS IN CHEMISTRY, Theodore E. Eckert, Department of Science, Whitney Point Central High School, Whitney Point, N. Y., Harley K. Lyons, Department of Science, Walnut Hills High School, Cincinnati, Ohio; and Wallace H. Strevell, Supervising Principal, Ellenville Public Schools, Ellenville, N. Y. Paper. Pages ii+158, 19×26 cm. 1947.

This is a combination laboratory manual and workbook which the busy teacher will find valuable as an aid to better teaching. It lists the conventional experiments, problems, and tables. The laboratory diagrams are good and clearly understandable. A good conventional laboratory and workbook manual.

KENNETH E. ANDERSON

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APPLIED CHEMISTRY, by Sherman R. Wilson, *Head of the Exact Science Department, Northwestern High School, Detroit, Michigan;* and Mary R. Mullins, *Chairman of Physical Science, Midwood High School, Brooklyn, New York.* Cloth. Pages xxi+714, 14×21 cm. 1947. Henry Holt and Company, New York, N. Y. Price $2.36.

This is a highly illustrated, non-technical book for the high school student who will become a consumer of the things of science. This book is an attempt to bring chemistry down to the level of the average student and acquaint him with the things he will need to know to be an intelligent consumer. It is an attempt to give students an understanding of how chemists are changing the world in which we live.

The experiments are simple and easy to follow. Many of the experiments are of a practical nature such as "How can I make cold cream?" or "How can I test my tooth powder to see if it is safe to use on my teeth?" The book abounds in experiments of this kind which students will enjoy performing. Each of the units has a "Summary of Main Ideas" and a list of "Review Questions." The teacher will find the list of "Examination Questions" at the end of the book very useful for review purposes. All in all it is a textbook worth considering.

KENNETH E. ANDERSON

DOORWAYS TO SCIENCE, by George W. Hunter, Ph.D., *Lecturer in Methods of Science Education, the Graduate School Associated Colleges at Claremont, Claremont, California,* and Walter G. Whitman, A.M., *Formerly Head, Physical Science Department, State Teachers College, Salem, Massachusetts.* Cloth. Pages xiii+546. 15.5×22 cm. 1947. American Book Company, 88 Lexington Avenue, New York 16, N. Y. Price $2.40.

Doorways to Science is an example of what can be done in modern school text writing by means of a fresh approach toward the whole undertaking. When consideration is given to worthwhile activities in and out of the classroom, to the motivations that click with modern youth, to the topics that stimulate and are vital in modern society, and the fruits of these considerations used to establish the framework and body of a general science text, then a new, valuable aid to teacher and student is created.

Doorways to Science represents such a departure from the traditional, formal general science text of the past. Its outstanding section is the unit on hobbies. Six hobbies are described in 41 pages under the unit title "Things to Do Outside of Class Time."

Hobby 1 How can you organize and run a science club?
Hobby 2 What is in the sky above us?
Hobby 3 How can I make good pictures?
Hobby 4 Do you enjoy collecting?
Hobby 5 Are you interested in living things?
Hobby 6 What are some interesting vacation hobbies?

This unit, the last in the text, closes with the topic "How May a Science Hobby Help Me After I Leave School?" This section discusses choice of vocation, the relation of hobbies to vocations, abilities needed for various

vocations, opportunities in various vocations, and changes of vocational choice. The unit on hobbies is aimed to meet one of the major objectives of general science-vocational guidance.

The book contains 256 modern photographs of excellent quality, besides an adequate number of explanatory diagrams. Each topic is followed with questions for class discussion, best answer quizzes, and new terms; each unit is followed with "Things to Remember in this Unit," scientific attitudes exercise, test on fundamental concepts, and a wealth of student activities. Preceding every unit is a page which ties the topic in with the student's personal life and experience.

The authors frankly advocate a functioning science club as an aid to teaching throughout the year. In this connection, the section on the science club in the unit on hobbies should be introduced early. Such activities as the excursion and the field trip become an integral part of the course. The best kind of teaching in this modern age is the kind that molds and inspires the lives of students by worthwhile, stimulating activities inside and outside of the classroom.

The content of *Doorways to Science* is as broad and deep as any text in general science. Modern texts would be deplorably lacking were they to give less treatment to atomic energy than is found in this work. Neptunium, plutonium, use of graphite, and absorption of slow neutrons by U-238 to produce new, unstable elements are all discussed here. For background in this unit, the Smyth Report should be "must reading" for all general science teachers. FM, radar, Loran, television, jet propulsion, and penicillin are some of the newer topics treated in this text. Under "Conservation and You," social significance is given to the following six problems: How is land wasted and what is being done to protect it?; How are forests used, abused, and protected?; How do we use water and how is it conserved? Why should we conserve our minerals?; How is our wild life protected and conserved?; How is human life conserved?

Considering content, illustration, motivation, and pedagogical planning, the book is as teachable and as stimulating as one could desire.

SHELDON S. MYERS
Western State High School
Western Michigan College
Kalamazoo, Michigan

SCIENCE, A STORY OF DISCOVERY AND PROGRESS, by Ira C. Davis, *Professor in the Teaching of Science, School of Education, University of Wisconsin, Head of Department of Science, University High School*, and Richard W. Sharpe, *Formerly Instructor in Science, George Washington High School, New York City*. Cloth. Pages xiii+538. 15×23.5 cm. 1947. Henry Holt and Company, Inc., 257 Fourth Avenue, New York, N. Y. Price $2.36.

It is the opinion of the reviewer that a single, best text does not exist for general science. Each teacher and school should select the text best suited for their philosophy and needs. On the one hand is the teaching style and temperament of the teacher; on the other hand are the needs of the boys and girls. The best choice of text is one in which these factors are carefully weighed. Just as we must not standardize methodology, so we must not standardize texts.

In the case of some teachers and classes, the text by Davis and Sharpe may well be the proper choice when the above factors are considered. Although containing material on such modern developments as atomic energy, radar, television, and penicillin, it is essentially an older, formal

text with modern additions. The work is abundantly illustrated with a total of 403 line diagrams for explanations. These diagrams amply supplement 167 photographs of inferior quality.

The Davis and Sharpe book is detailed and comprehensive with rather elaborate treatment of such topics as light and sound. The section on light resembles that of regular physics, While lacking many of the modern motivations and activities, the text will supply dilligent classes with a broad and deep background in general science.

The topics associated with each unit are of the ideal length for teaching assignments. Demonstrations, pupil activities, and review questions are set apart from the text in smaller type. New words are defined in the text along with phonetic spellings. The four to seven topics in each unit are arranged in order of difficulty, enabling some differentiation of work according to student ability.

For ninth grade students, this book should have a stouter binding. However, there is always the argument that any book ought to last a normal life with proper student care. For many schools of limited budgets, the durability of a book becomes an important factor in its choice. Few books reaching the market these days have bindings anywhere near adequate for adolescent exuberance and abandonment.

SHELDON S. MYERS

ATOMICS FOR THE MILLIONS, by Dr. Maxwell Leigh Eidinoff, Department of Chemistry, Queens College; and Hyman Ruchlis, Lafayette High School, Brooklyn, New York. Cloth. Pages xiv+281. 15×23 cm. 1947. Whittlesey House, McGraw-Hill Book Company, Inc., New York, N. Y. Price $3.50.

Here is really a book for those who are not physicists or physics students' but many teachers of physics will read it with profit and pleasure. It starts far back in the history of science but in a few pages covers the contributions of Democritus, Aristotle, Cavendish, Priestly, Proust, Dalton, and others who had the early concepts of the atomic structure. The investigations of Newton, Rumbord, Davy, Brown, Franklin, Galvani, Volta, and Maxwell form the basis of Chapter three. This is followed by the investigations of Crookes, Thomson, and Millikan on vacuum tubes; then the work of Roentgen, Becquerel, Newton, Huygens, Herschel, Hertz, and Planck on the relation of light and electricity. In fifty pages of simple narration and explanation the authors have told the story so that anyone may read and understand.

Part II is the *Architecture*. Here the great names are Madam Curie, Rutherford, C. T. R. Wilson and the topic alpha, beta and gamma rays and natural radioactivity. Next follows the later work of Rutherford, and that of Neils Bohr in working out the atomic pattern, assisted by such men as Aston with his mass spectrometer and Richards in his accurate determination of atomic weight. Great *atom guns* come next—Van de Graaff with his electrostatic generator, Lawrence and the cyclotron, Kerst and the betatron—giant machinery to investigate the smallest of particles. Modern alchemy now becomes possible and Einstein explains with $E=mc^2$, but in language you can understand. New particles are found: the neutron, the deuteron, the positron. Millikan and Compton make separate investigations of cosmic rays, and the meson becomes another puzzle. Next is artificial radioactivity first accomplished by the Joliot's in 1934.

We have now covered half the book and are ready for the great story of the release of atomic energy. No attempt will be made to review the next eighty pages. You must read the story of the development of the atomic bomb and the work of the four major groups of the world's outstanding

scientists and their many assistants at the Argonne Laboratory, the Clinton Laboratory, at Hanford, and at Alamogordo. Also read the story of the first test in the New Mexico desert, the destruction of the cities of Hiroshima and Nagasaki, and the scientific tests at Bikini.

Part IV tells of the possible uses that may come from the development of atomic energy and its control in the world of the future when its destructive use has been brought under control of a united world power. Much of this is of course largely visionary but it is based on the facts now known and the progress of the recent past.

G. W. W.

ON UNDERSTANDING SCIENCE, AN HISTORICAL APPROACH, by James B. Conant, *President of Harvard University*. Cloth Pages xiii + 145. 13 × 20.5 cm. 1947. Yale University Press, New Haven, Conn. Price $2.00.

Is it possible for the college educated man to understand the modern scientist without first majoring in science? Here President Conant gives his views on a type of science course for all non-science college students showing how the great advances in science have developed, the errors made by some of the great experimenters, and the long time required for a lucky guess or a carefully thought out proposition to become established as a scientific truth. His plan is different from that of many who have attempted to build courses for the general student. He says: "It is therefore to observation and experiment and the less spectacular and less known advances in the sciences that I would primarily direct attention. More can be learned from their study than from the few famous sweeping generalizations which in their time gathered up so much of the past and illuminated the future so brightly."

Two important topics are used by the author to show his plan for this study: first, in the seventeenth century, the study of water pumps, and the pressure exerted by the air which led to the development of Boyle's Law; second, in the eighteenth century, the study of animal electricity by Galvani and his followers and the construction of the electric battery by Volta, then next the study of combustion leading from Priestly and phlogiston to the new concepts of Lavoisier. In this manner the author "outlines a method of approach" which skillful teachers may try out. He sees on every hand "renewed interest in experimentation with new methods of teaching science as part of a general education."

G. W. W.

PREPARATORY BUSINESS MATHEMATICS, by Lloyd L. Small, Professor of Mathematics, Lehigh University. Cloth. Pages x + 244. 14.5 × 21.5 cm. 1947. $2.75. The Ronald Press Company, 15 E 26th St. New York, N. Y.

According to the author's preface this book is designed to be used as a first course in college mathematics for majors in Business Administration. It includes the basic material for all later courses in business mathematics and omits some of the traditional college algebra which is of little value to the business student.

The text covers, elementary algebraic operations, linear equations, powers and roots, logarithms, arithmetic and geometric progressions, functions and graphic representation, permutations and combinations. the binomial theorem, and probability in about the same manner as they are given in a first semester course in college algebra. However more applications to business situations are used. The chapter on quadratic equations is brief and the problems are not especially applicable to business. Significant

digits, relative error and approximate computation are adequately discussed. The subject of ratio and proportion is almost omitted.

Chapter VIII on, "Certain Important Types of Graphs," gives a very good analysis of the standard curves including logarithmic and exponential along with tests for them. Calculus as such is not introduced but successive differences are used to show relations of variables in first and second degree equations. Chapter IX, "Curve Fitting," is a good explanation of fitting raw data to some of the standard curves. The problems of this chapter are not as applicable to business situations as they might be. The chapter on logarithms is very complete and the applications appropriate. The pattern of the text is, a statement of the principle in italics, illustrative examples with explanations and then lists of exercises. There are over 650 exercises many of them similar to those found in a college algebra. Answers are given, the book is well indexed, has 4 place logarithm tables, a table of squares and square roots and an American Experience Table of Mortality.

The book is well written and does place emphasis on the use of mathematics in business. However I do not feel that the difference between this text and a college algebra is very great but no doubt it is a step ahead to a closer correlation of principle and application.

PHILIP PEAK
Bloomington, Ind.

PLANE TRIGONOMETRY by Elmer B. Mode, Professor of Mathematics, Boston University, Cloth. Pages 200. 14×21 cm. 1947. Prentice-Hall, Inc. 70 5th Ave. New York, N. Y. $2.40.

This text is based on a lithoprinted work formerly used and published by the author. It covers the standard material of plane trigonometry but has organized it a little differently and gives emphasis to different items as will be shown in the following comments. Throughout the text attention is called to the use of significant numbers, the part they play in actual practice and how they fit into the author's particular problems. The general explanation of approximate computation is given as a supplementary chapter at the end of the book. It seems to the reviewer this phase of the text is very well done. The slide rule and logarithms are also brought in as supplementary chapters, which leaves the context of the book strictly trigonometry. Very good use is made of historical material. An excellent explanation is given concerning the inability to define the tan. 90° and cot 0° along with the principle of approaching ∞ as θ approaches a limit.

Chapter V entitled, "Miscellaneous Topics," gives brief but good introductions to some of the relationships of trigonometry with physical principles of sound, harmonic motion, power series, Fourier series etc. These add greatly to the appreciation of the power of trigonometry. Line drawings and pictorial illustrations are good. Radian measure and mil measure are both included but used very little in the text. Chapter IV gives the derivation of the reduction formulas as based on reference triangles which are congruent. This clarifies the concept a great deal. There are many verbal problems coming from a number of subjects, located within the chapter and with sufficient explanation to make the problem meaningful. Tables included are: a combination of natural and trigonometric functions, natural and Briggs logarithms, table of squares, conversion tables and a list of useful constants. The tables are four place tables. Answers are given to the odd numbered problems and are available for the even. Throughout the text the author refers to preceding sections for further meaning and clarification thereby aiding in the continunity of the subject.

PHILIP PEAK

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